

# THE NINETY-TWO JOHNSON SOLIDS





## **Mathematician Norman Johnson was born in Chicago in 1930.**

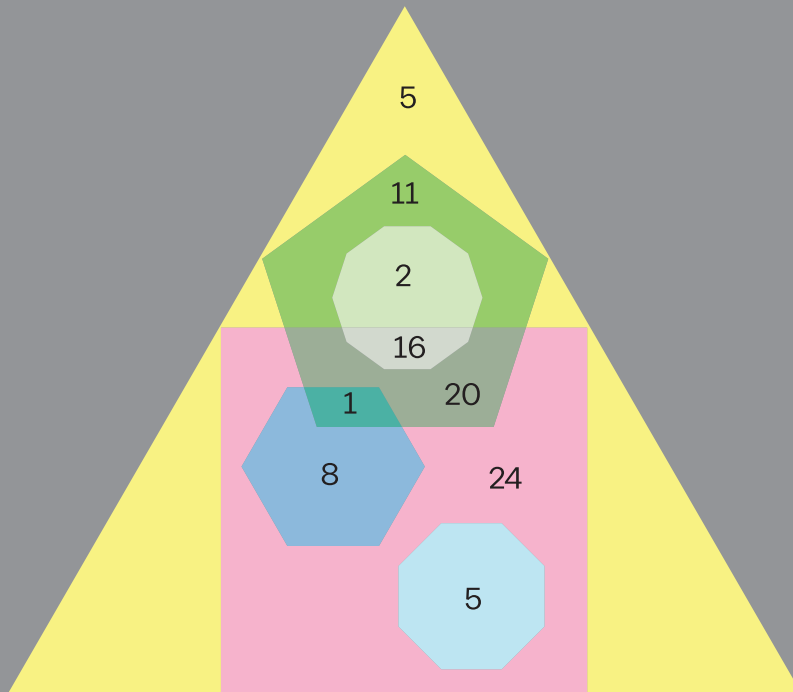
In 1966, he identified 92 specific non-uniform convex solids, all with regular faces. He speculated that no more could exist. In 1969, it was proven by mathematician Victor Zalgaller that Johnson's theory was correct. A total of exactly 92 solids exist. These polyhedra became known as the "Johnson Solids."

After earning his PhD from the University of Toronto, Johnson went on to hold a position at the Mathematics Department of Wheaton College in Massachusetts until his retirement in 1998. His PhD supervisor was the great geometer H.S.M. Coxeter, known for his 1973 book *Regular Polytopes*, which describes the properties of perfect solids through higher dimensions.

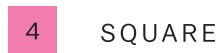
Norman Johnson died on July 13, 2017.

Just before his death, he completed the book *Geometries and Transformations*, which was published by Cambridge University Press in 2018.

The 92 Johnson solids are convex polyhedra whose faces are a combination from six different regular polygon shapes. It turns out that only these shapes fit together in a certain number of ways to produce the polyhedra. The polygon faces must be “perfect,” meaning they have symmetry and same-sized sides: an equilateral triangle, a perfect square, and so on. Seven-sided polygons, for example, will not work. There are no Johnson solids composed of all six of the shapes. There are five Johnson solids composed of only triangles. This Venn diagram maps the number of shape combinations that comprise the 92 polyhedra. In the following pages, you will find the Johnson solids organized into groups according to their shapes. On the facing page is a summary.



## NUMBER OF SIDES



## NUMBER OF JOHNSON SOLIDS

$$\triangle = 5$$

$$\triangle + \square = 24$$

$$\triangle + \pentagon = 11$$

$$\triangle + \square + \pentagon = 20$$

$$\triangle + \square + \hexagon = 8$$

$$\triangle + \square + \octagon = 5$$

$$\triangle + \pentagon + \text{decagon} = 2$$

$$\triangle + \square + \pentagon + \hexagon = 1$$

$$\triangle + \square + \pentagon + \text{decagon} = 16$$

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TOTAL = 92

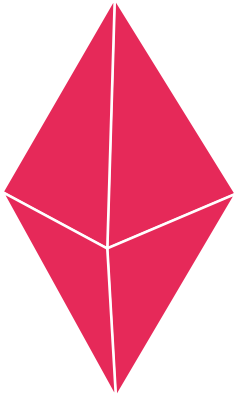
square pyramid **§** pentagonal pyramid **§** triangular cupola **§** square cupola **§** pentagonal cupola **§**  
pentagonal rotunda **§** elongated triangular pyramid **§** elongated square pyramid **§** elongated pentagonal  
pyramid **§** gyroelongated square pyramid **§** gyroelongated pentagonal pyramid **§** triangular bipyramid  
**§** pentagonal bipyramid **§** elongated triangular bipyramid **§** elongated square bipyramid **§** elongated  
pentagonal bipyramid **§** gyroelongated square bipyramid **§** elongated triangular cupola **§** elongated  
square cupola **§** elongated pentagonal cupola **§** elongated pentagonal rotunda **§** gyroelongated  
triangular cupola **§** gyroelongated square cupola **§** gyroelongated pentagonal cupola **§** gyroelongated  
pentagonal rotunda **§** gyrobifastigium **§** triangular orthobicupola **§** square orthobicupola **§** square  
gyrobicupola **§** pentagonal orthobicupola **§** pentagonal gyrobicupola **§** pentagonal orthocupolarotunda  
**§** pentagonal gyrocupola rotunda **§** pentagonal orthobiotunda **§** elongated triangular orthobicupola  
**§** elongated triangular gyrobicupola **§** elongated square gyrobicupola **§** elongated pentagonal  
orthobicupola **§** elongated pentagonal gyrobicupola **§** elongated pentagonal orthocupolarotunda  
**§** elongated pentagonal gyrocupolarotunda **§** elongated pentagonal orthobiotunda **§** elongated  
pentagonal gyrobirotunda **§** gyroelongated triangular bicupola **§** gyroelongated square bicupola **§**  
gyroelongated pentagonal bicupola **§** gyroelongated pentagonal cupolarotunda **§** gyroelongated  
pentagonal birotunda **§** augmented triangular prism **§** biaugmented triangular prism **§** triaugmented  
triangular prism **§** augmented pentagonal prism **§** biaugmented pentagonal prism **§** augmented  
hexagonal prism **§** parabiaugmented hexagonal prism **§** metabiaugmented hexagonal prism **§**  
triauxmented hexagonal prism **§** augmented dodecahedron **§** parabiaugmented dodecahedron **§**  
metabiaugmented dodecahedron **§** triaugmented dodecahedron **§** metabidiminished icosahedron **§**  
tridiminished icosahedron **§** augmented tridiminished icosahedron **§** augmented truncated tetrahedron  
**§** augmented truncated cube **§** biaugmented truncated cube **§** augmented truncated dodecahedron **§**  
parabiaugmented truncated dodecahedron **§** metabiaugmented truncated dodecahedron **§** triaugmented  
truncated dodecahedron **§** gyrate rhombicosidodecahedron **§** parabigyrate rhombicosidodecahedron  
**§** metabigyrate rhombicosidodecahedron **§** trigyrate rhombicosidodecahedron **§** diminished  
rhombicosidodecahedron **§** paragyrate diminished rhombicosidodecahedron **§** metagyrate diminished  
rhombicosidodecahedron **§** bigyrate diminished rhombicosidodecahedron **§** parabidiminished  
rhombicosidodecahedron **§** metabidiminished rhombicosidodecahedron **§** gyrate bidiminished  
rhombicosidodecahedron **§** tridiminished rhombicosidodecahedron **§** snub disphenoid **§** snub square  
antiprism **§** sphenocorona **§** augmented sphenocorona **§** sphenomegacorona **§** hebesphenomegacorona  
**§** disphenocingulum **§** bilunabiotunda **§** triangular hebesphenorotunda

THE SOLIDS

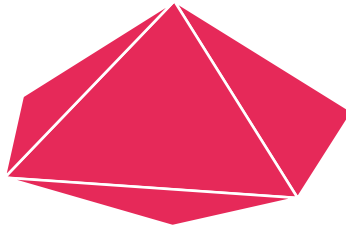
The image features a minimalist, abstract design. It consists of several overlapping geometric shapes, primarily triangles, set against a light blue background. A large, vibrant red triangle is positioned on the left side, pointing towards the bottom-left corner. Overlapping its right edge is a large, soft pink triangle that extends towards the top-right corner. The top edge of the composition is a sharp, narrow point where the pink triangle meets the light blue background. The overall aesthetic is clean and modern, with a focus on color and geometric form.

## Triangle faced

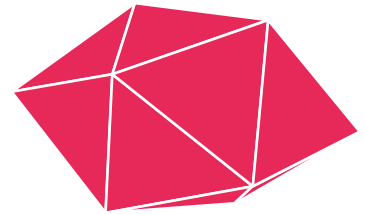
Five solids are called *deltahedra*, meaning they are made of only triangles. Johnson solid triangles are always equilateral.



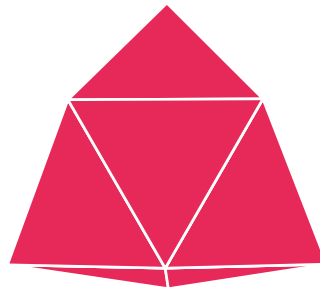
J12  
TRIANGULAR BIPYRAMID



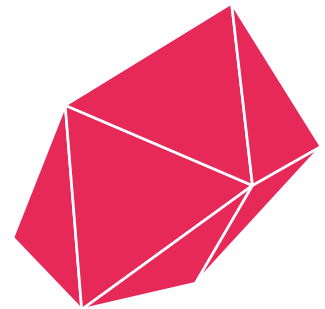
J13  
PENTAGONAL BIPYRAMID



J17  
GYROELONGATED  
SQUARE BIPYRAMID



J51  
TRIAUGMENTED  
TRIANGULAR PRISM

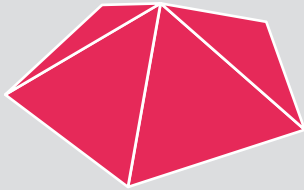


J84  
SNUB DISPHENOID



## Triangle + Pentagon faced

Eleven solids have only triangle and pentagon faces.



J2  
PENTAGONAL PYRAMID



J11  
GYROELONGATED  
PENTAGONAL PYRAMID



J60  
METABIAUGMENTED  
DODECAHEDRON



J34  
PENTAGONAL  
ORTHOBIROTUNDA



J48  
GYROELONGATED  
PENTAGONAL  
ORTHOBIROTUNDA



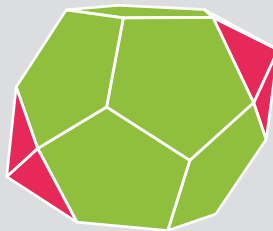
J61  
TRIAUGMENTED  
DODECAHEDRON



J62  
METABIDIMINISHED  
ICOSAHEDRON



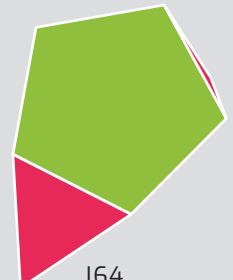
J58  
AUGMENTED  
DODECAHEDRON



J59  
PARABIAUGMENTED  
DODECAHEDRON



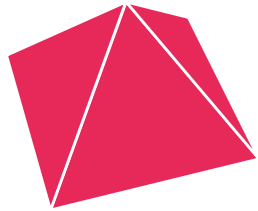
J63  
TRIDIMINISHED  
ICOSAHEDRON



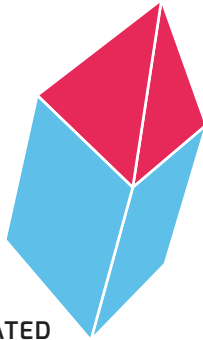
J64  
AUGMENTED  
TRIDIMINISHED  
ICOSAHEDRON

# Triangle + Square faced

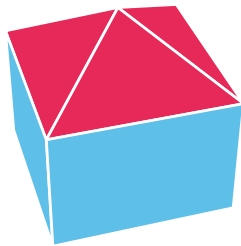
Twenty-four solids have faces made up of only triangles and squares.



J1  
SQUARE PYRAMID



J7  
ELONGATED  
TRIANGULAR PYRAMID

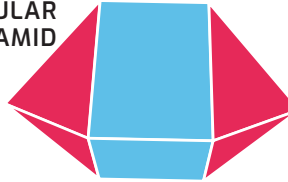


J8  
ELONGATED  
SQUARE  
PYRAMID

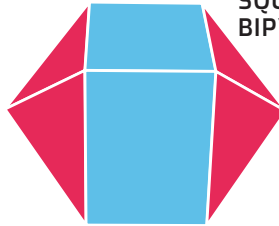


J10  
GYROELONGATED  
SQUARE PYRAMID

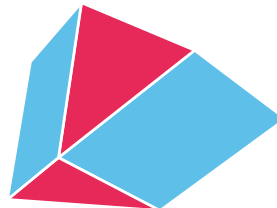
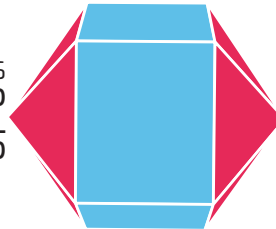
J14  
ELONGATED  
TRIANGULAR  
BIPYRAMID



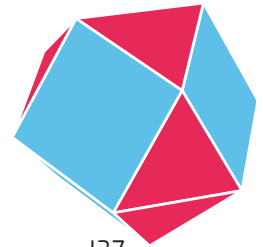
J15  
ELONGATED  
SQUARE  
BIPYRAMID



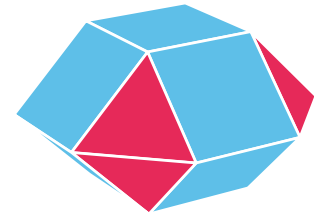
J16  
ELONGATED  
PENTAGONAL  
BIPYRAMID



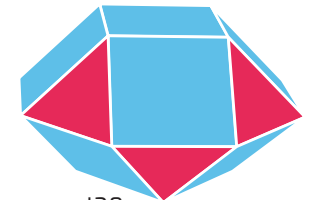
J26  
GYROBIFASTIGIUM



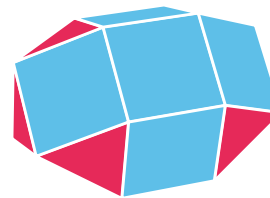
J27  
TRIANGULAR  
ORTHOBICUPOLA



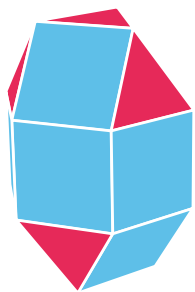
J28  
SQUARE  
ORTHOBICUPOLA



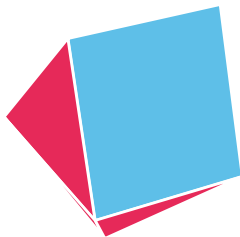
J29  
SQUARE  
GYROBICUPOLA



J35  
ELONGATED  
TRIANGULAR  
ORTHOBICUPOLA



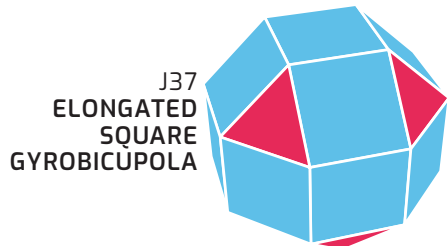
J36  
ELONGATED  
TRIANGULAR  
GYROBICUPOLA



J49  
AUGMENTED  
TRIANGULAR PRISM



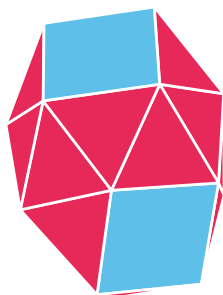
J88  
SPHENOMEGACORONA



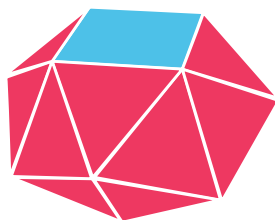
J37  
ELONGATED  
SQUARE  
GYROBICUPOLA



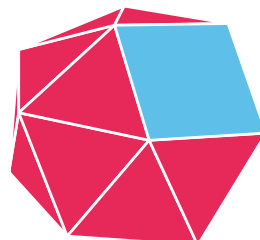
J50  
BIUGMENTED  
TRIANGULAR PRISM



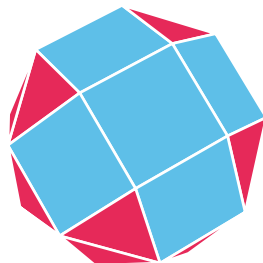
J44  
GYROELONGATED  
TRIANGULAR  
BICUPOLA



J85  
SNUB SQUARE  
ANTIPRISM



J89  
HEBESPHENOMEGACORONA



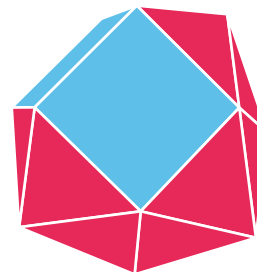
J45  
GYROELONGATED  
SQUARE BICUPOLA



J86  
SPHENOCORONA



J87  
AUGMENTED  
SPHENOCORONA



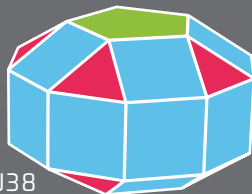
J90  
DISPHENOCINGULUM

# Triangle + Square + Pentagon faced

Twenty solids have only triangle, square and pentagon faces



J9  
ELONGATED  
PENTAGONAL  
PYRAMID



J38  
ELONGATED  
PENTAGONAL  
ORTHOBIKUPOLA



J42  
ELONGATED  
PENTAGONAL  
ORTHOBIKUPOLA



J30  
PENTAGONAL  
ORTHOBIKUPOLA



J39  
ELONGATED  
PENTAGONAL  
GYROBIKUPOLA



J31  
PENTAGONAL  
GYROBIKUPOLA



J40  
ELONGATED  
PENTAGONAL  
ORTHOBIKUPOLAROTUNDA



J43  
ELONGATED  
PENTAGONAL  
GYROBIKUPOLA



J32  
PENTAGONAL  
ORTHOBIKUPOLAROTUNDA



J41  
ELONGATED  
PENTAGONAL  
GYROBIKUPOLAROTUNDA



J33  
PENTAGONAL  
GYROBIKUPOLAROTUNDA



J46  
GYROELONGATED  
PENTAGONAL  
BICUPOLA



J72  
GYRATE  
RHOMBICOSIDODECAHEDRON



J73  
PARABIGYRATE  
RHOMBICOSIDODECAHEDRON



J47  
GYROELONGATED  
PENTAGONAL  
CUPOLAROTUNDA

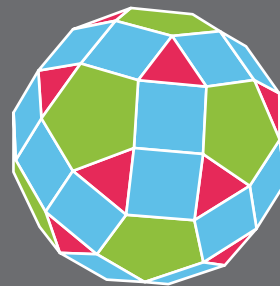


J74  
METABIGYRATE  
RHOMBICOSIDODECAHEDRON



J52  
AUGMENTED  
PENTAGONAL  
PRISM

J75  
TRIGYRATE  
RHOMBICOSIDODECAHEDRON



J53  
BIAUGMENTED  
PENTAGONAL  
PRISM

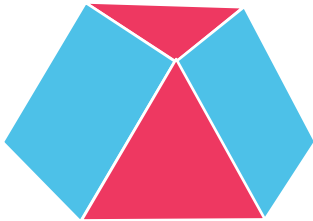


J91  
BILUNABIROTUNDA

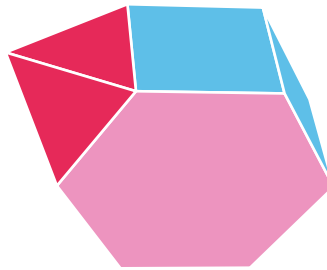


# Triangle + Square + Hexagon faced

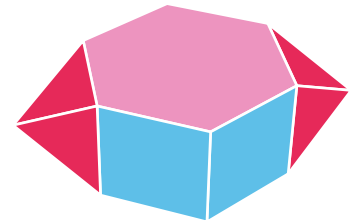
Eight solids have only triangle, square and hexagon faces



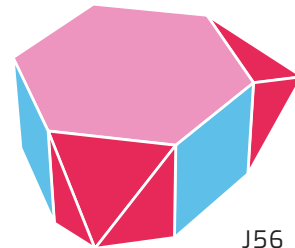
J3  
TRIANGULAR  
CUPOLA



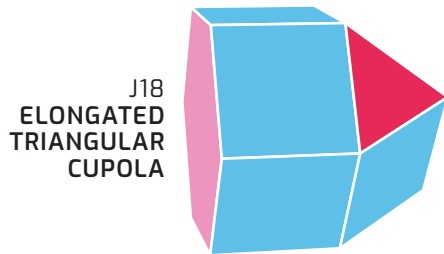
J54  
AUGMENTED  
HEXAGONAL  
PRISM



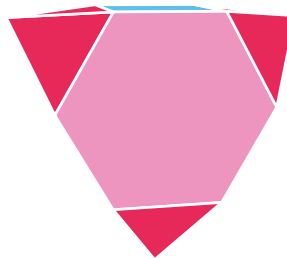
J55  
PARABIAUGMENTED  
HEXAGONAL  
PRISM



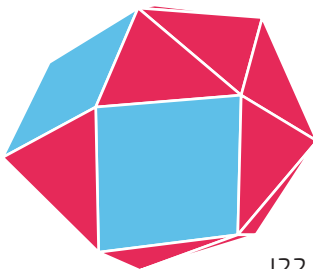
J56  
METABIAUGMENTED  
HEXAGONAL  
PRISM



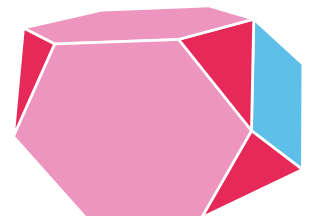
J18  
ELONGATED  
TRIANGULAR  
CUPOLA



J57  
TRIAUGMENTED  
HEXAGONAL  
PRISM



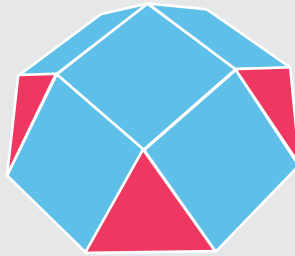
J22  
GYROELONGATED  
TRIANGULAR  
CUPOLA



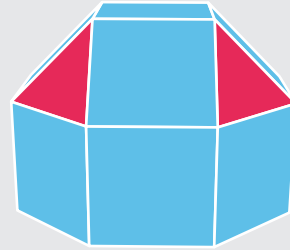
J65  
AUGMENTED  
TRUNCATED  
TETRAHEDRON

# Triangle + Square + Octagon faced

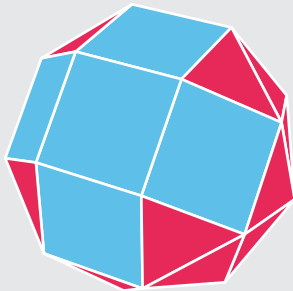
Five solids have only triangle, square and octagon faces



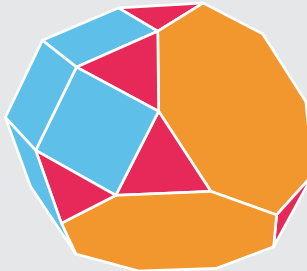
J4  
SQUARE CUPOLA



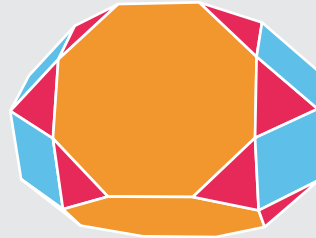
J19  
ELONGATED  
SQUARE CUPOLA



J23  
GYROELONGATED  
SQUARE CUPOLA



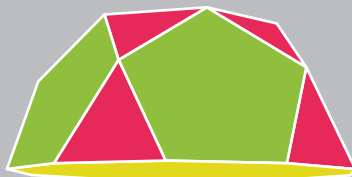
J66  
AUGMENTED  
TRUNCATED CUBE



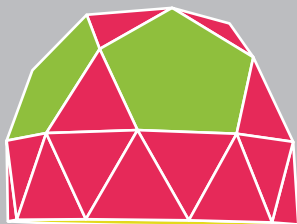
J67  
BIAUGMENTED  
TRUNCATED CUBE

## Triangle + Pentagon + Decagon faced

Two solids have only triangle, pentagon and decagon faces



J6  
PENTAGONAL  
ROTUNDA

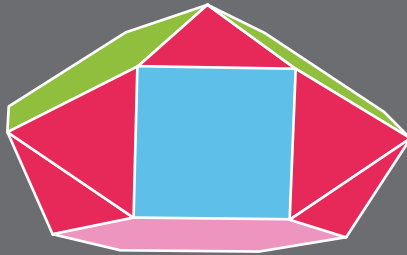


J25  
GYROELONGATED  
PENTAGONAL  
ROTUNDA



# Triangle + Square + Pentagon + Hexagon faced

Only one solid has faced composed of four different shape faces.

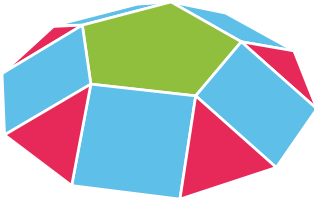


J92  
TRIANGULAR  
HEBESPHENOROTUNDA

# Triangle + Square + Pentagon + Decagon faced

Sixteen solids have only triangle, square, pentagon and decagon faces

J5  
PENTAGONAL  
CUPOLA



J68  
AUGMENTED  
TRUNCATED  
DODECAHEDRON



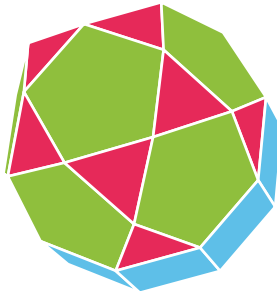
J20  
ELONGATED  
PENTAGONAL  
CUPOLA



J69  
PARABIAUGMENTED  
TRUNCATED  
DODECAHEDRON



J21  
ELONGATED  
PENTAGONAL  
ROTUNDA



J70  
METABIAUGMENTED  
TRUNCATED  
DODECAHEDRON



J71  
TRIAUGMENTED  
TRUNCATED  
DODECAHEDRON





J76  
DIMINISHED  
RHOMBICOSIDODECAHEDRON

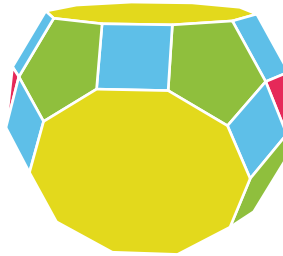
J80  
PARABIDIMINISHED  
RHOMBICOSIDODECAHEDRON



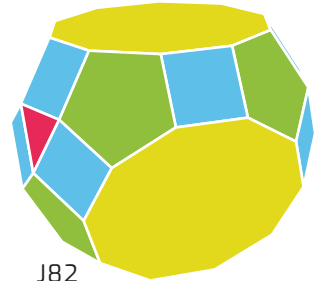
J81  
METABIDIMINISHED  
RHOMBICOSIDODECAHEDRON



J77  
PARAGYRATE  
DIMINISHED  
RHOMBICOSIDODECAHEDRON



J82  
GYRATE BIDIMINISHED  
RHOMBICOSIDODECAHEDRON



J78  
METAGYRATE  
DIMINISHED  
RHOMBICOSIDODECAHEDRON



J83  
TRIDIMINISHED  
RHOMBICOSIDODECAHEDRON



J79  
BIGYRATE  
DIMINISHED  
RHOMBICOSIDODECAHEDRON

