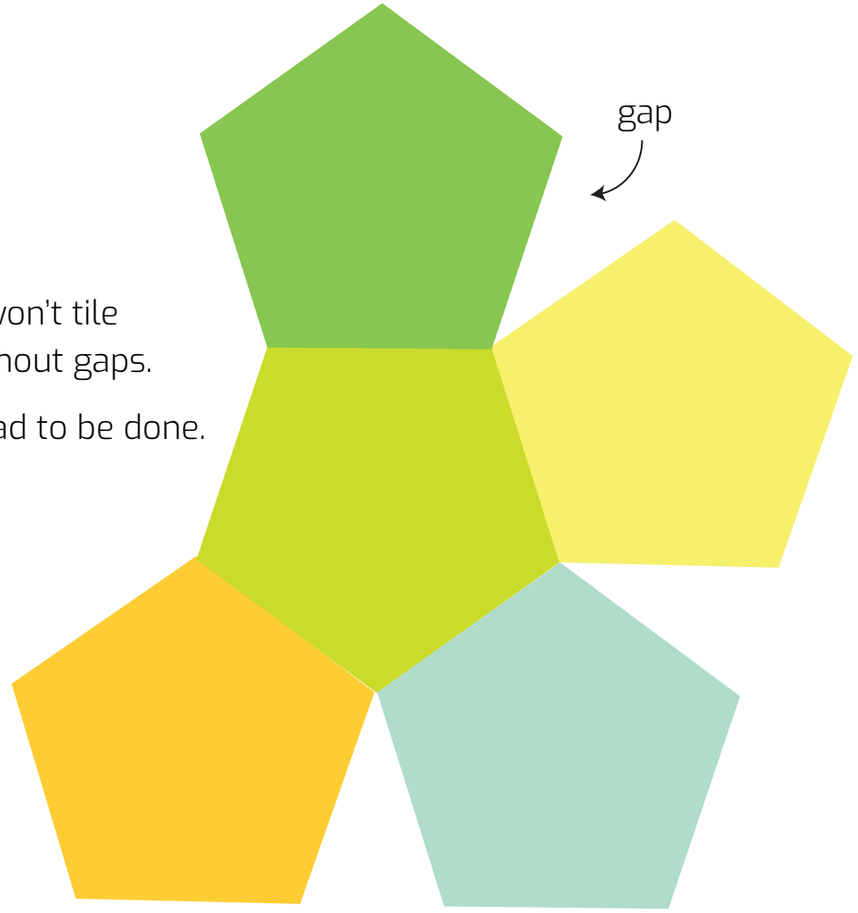
THE FIFTEEN
PENTAGONAL TESSELLATIONS
OF THE 2D PLANE

pointlineplane.info

A pentagon won't tile
the plane without gaps.

Something had to be done.

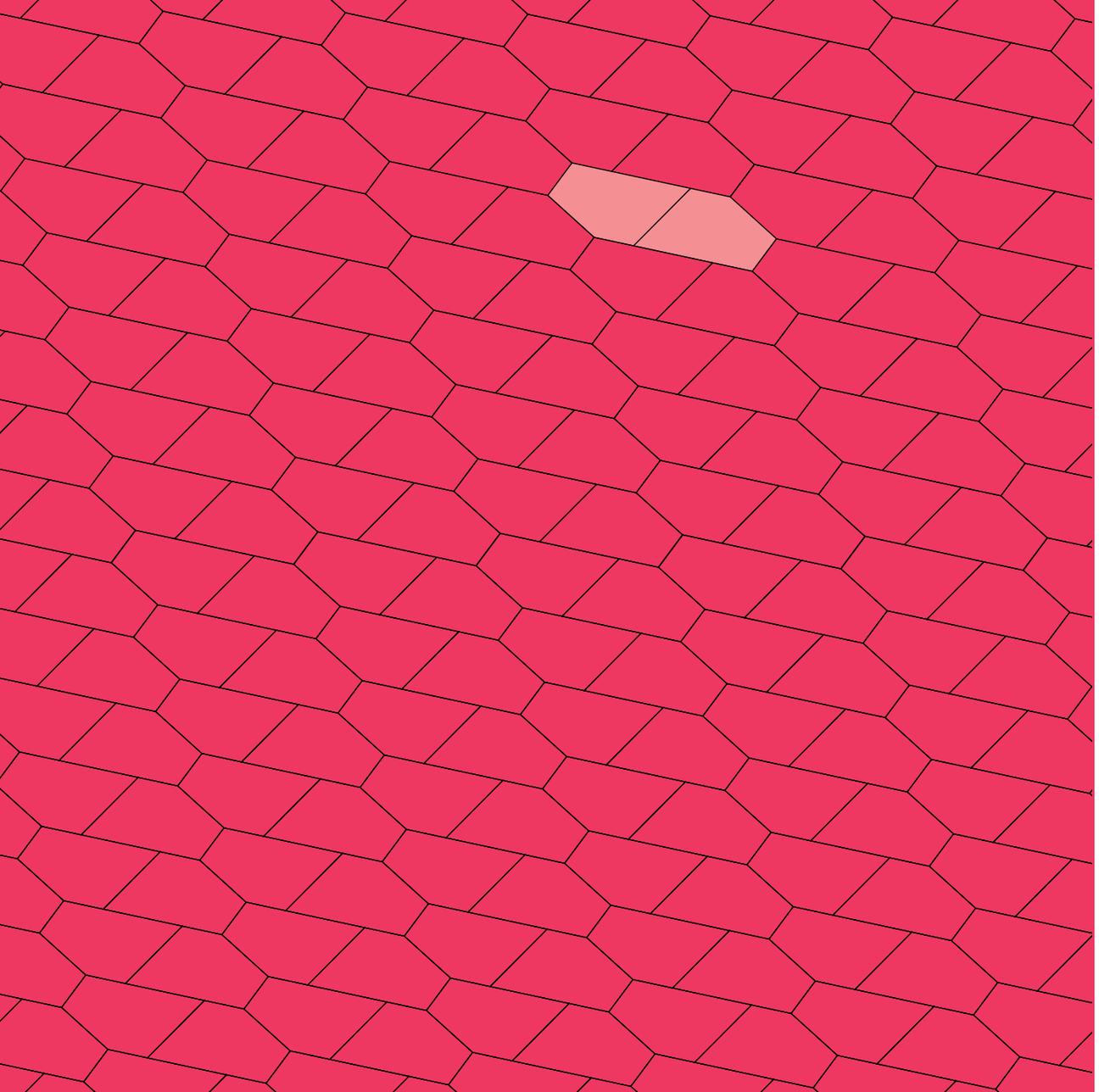


There are only 15 possible pentagons that will tile the plane.

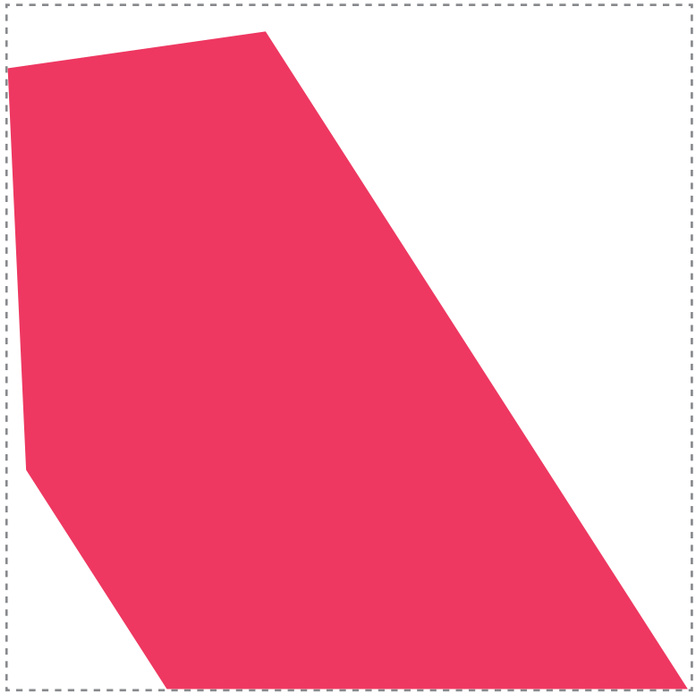
But at one time, nobody knew if pentagons could tessellate at all. If they could, how many varying pentagonal shapes are there?

In 1918, mathematician Karl Reinhardt decided to find them. He found five. Fifty years later, R. B. Kershner found 3 more and was satisfied that no more could be found. Then Richard E. James III found another one in 1975. In 1977 non-mathematician Marjorie Rice, known as the "San Diego housewife" found four more by working in secret at her kitchen table when her family slept. Then in 1985, Rolf Stein found the 14th. Finally, in 2015, a 15th irregular pentagon was discovered by husband and wife team Jennifer McCloud-Mann and Casey Mann, along with graduate student David von Derau. Two years later, in 2017, Michaël Rao proved that only 15 tilings exist. The search was over.

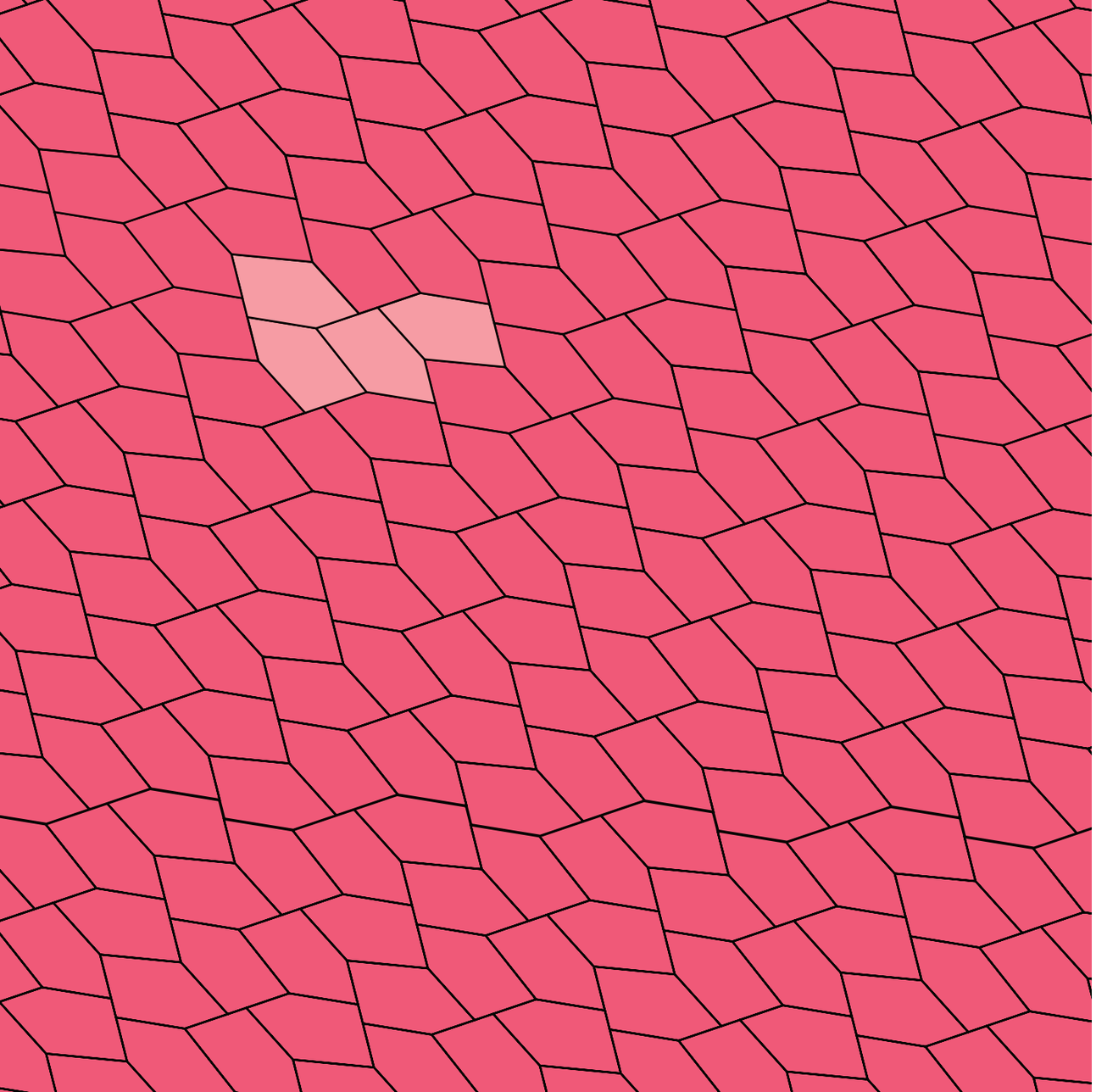
You might be surprised to learn that most of these tile types have flexible degrees of freedom, meaning their internal angles and edge lengths can vary infinitely. In other words, they are stretchable. Only Types 14 and 15 must maintain their rigid form to tesselate. In the tilings shown on the following pages, the highlighted tile groups represent each type's "primitive unit," the smallest possible section of the tiling that generates the whole tiling using only translations (repeats).



01



1918 / REINHARDT



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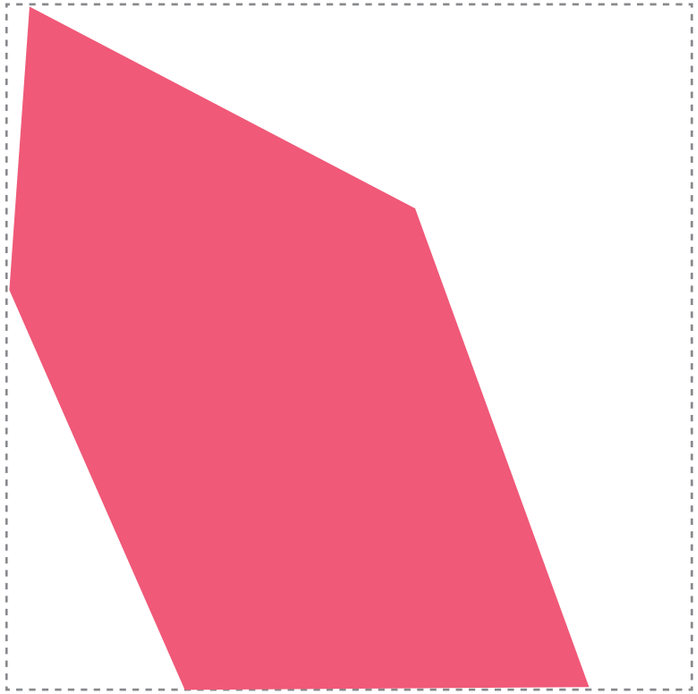
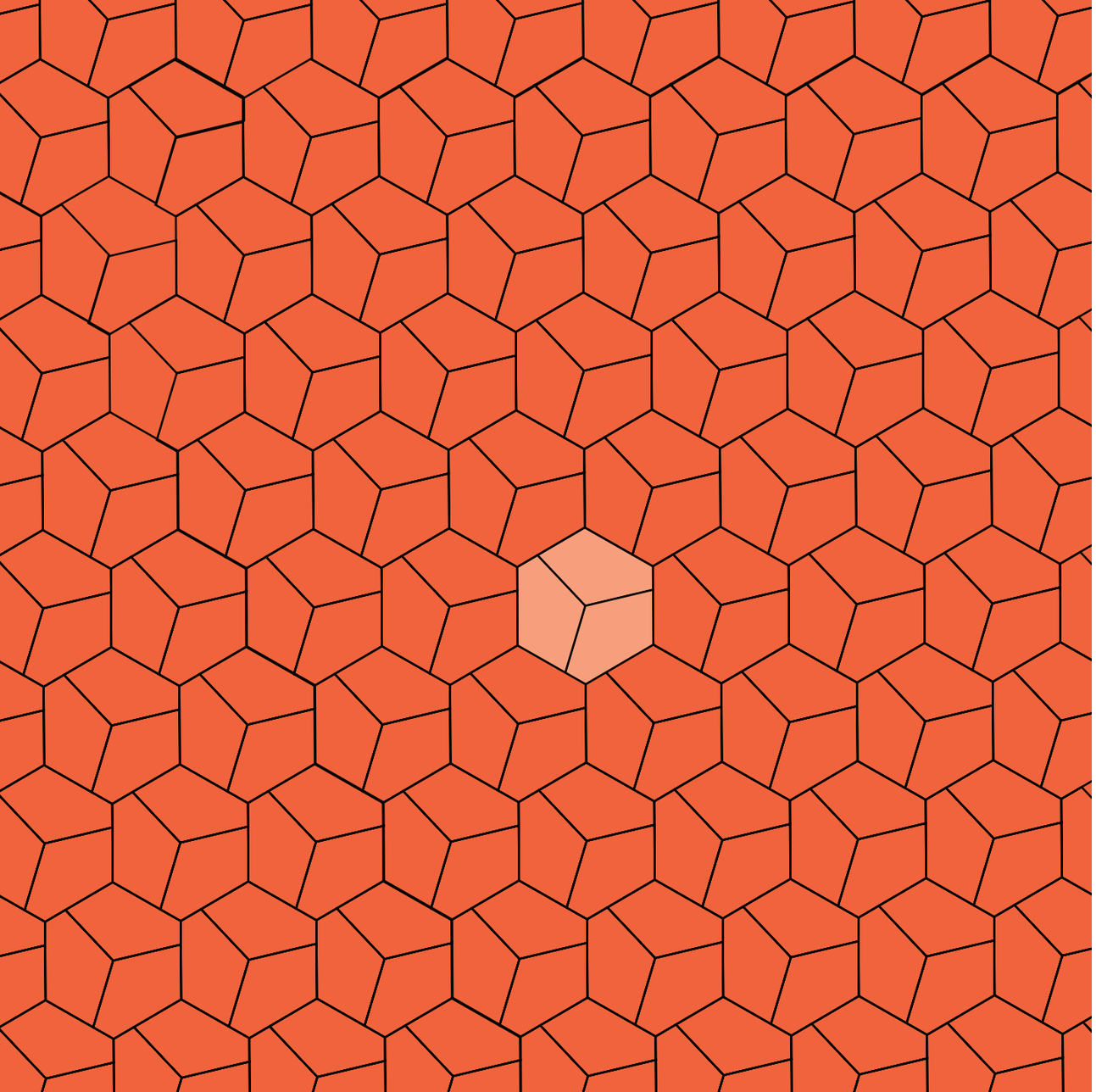


1918 / REINHARDT



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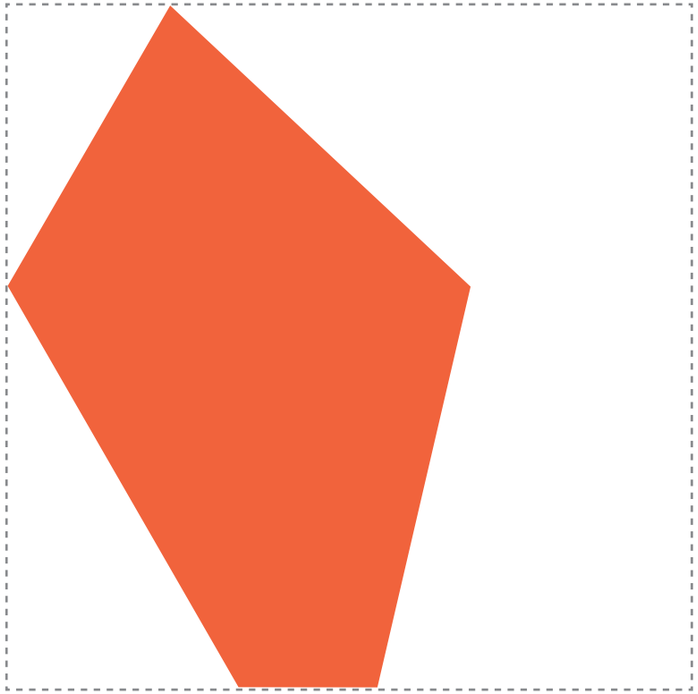
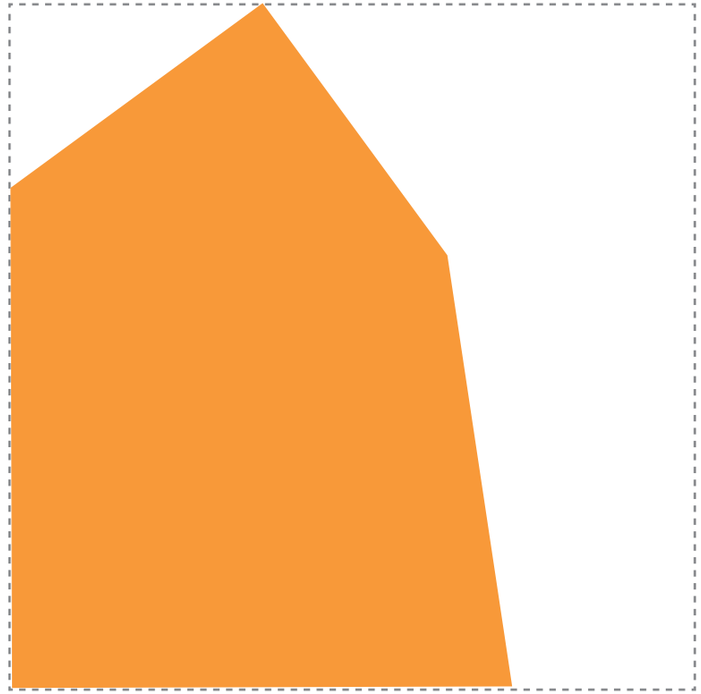


1918 / REINHARDT

04



1918 / REINHARDT

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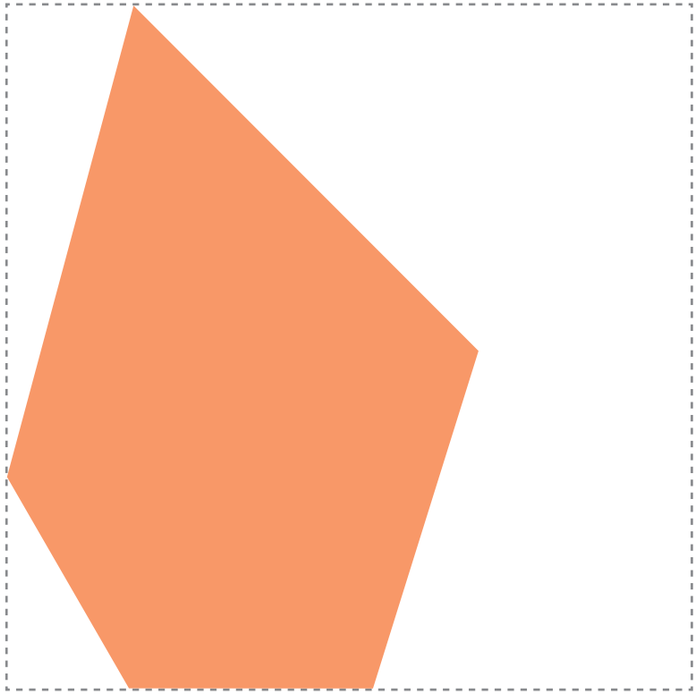
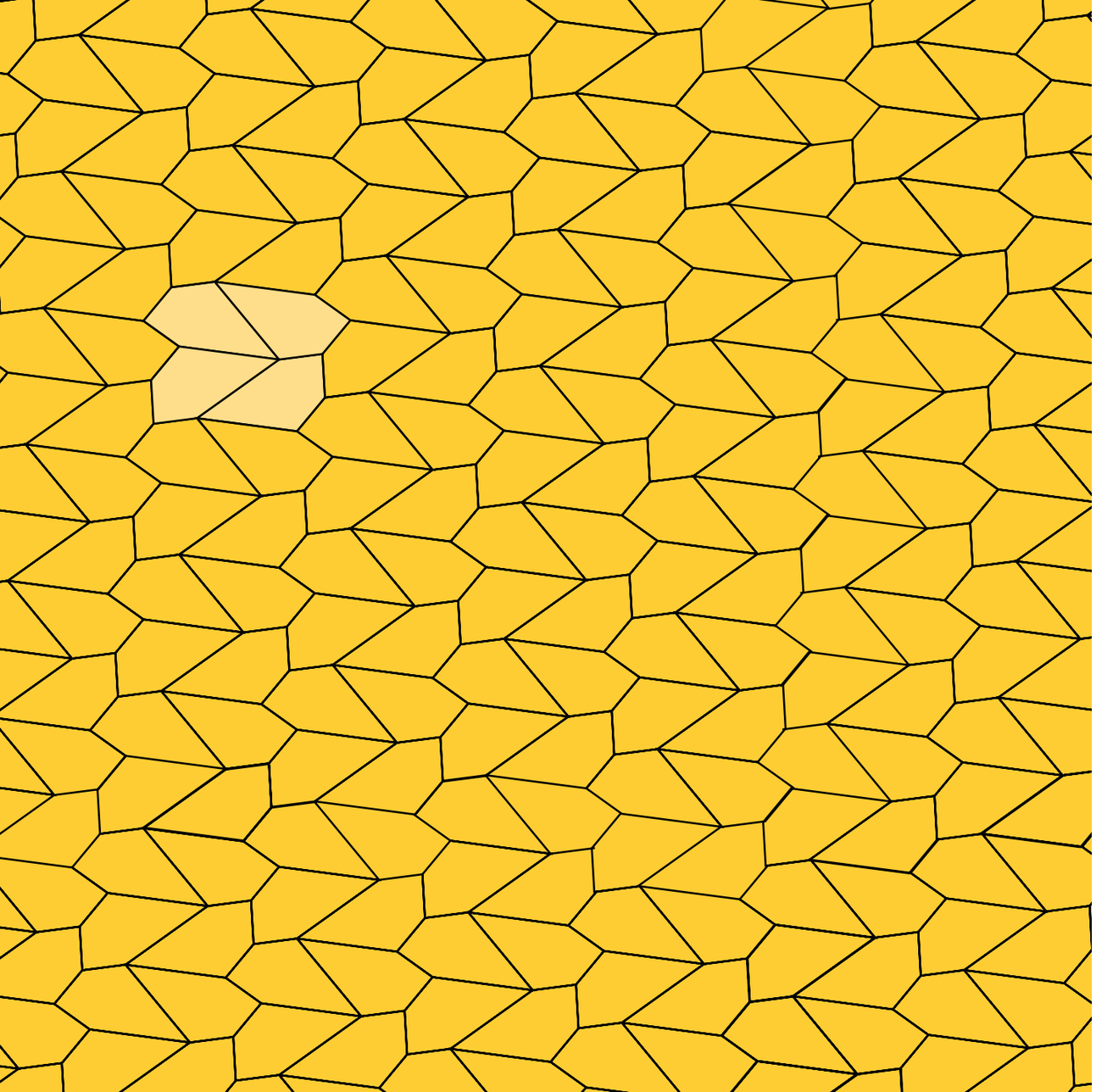


1918 / REINHARDT



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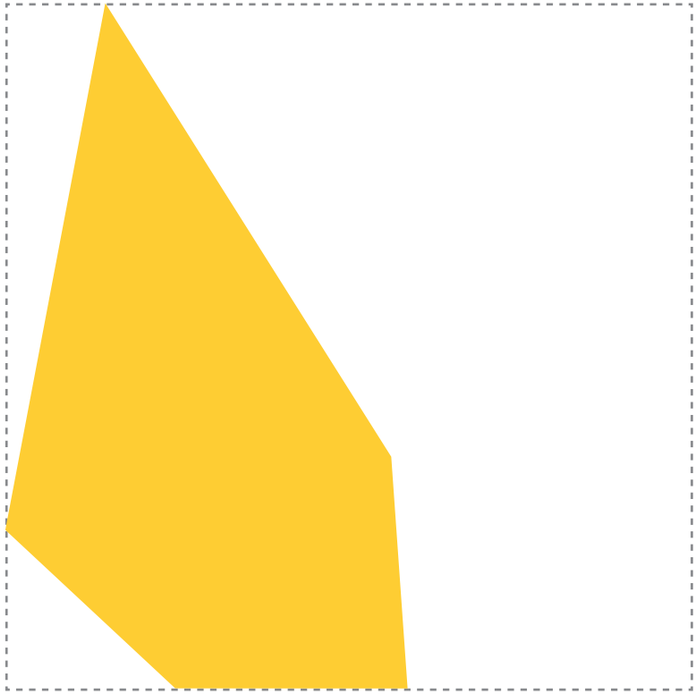
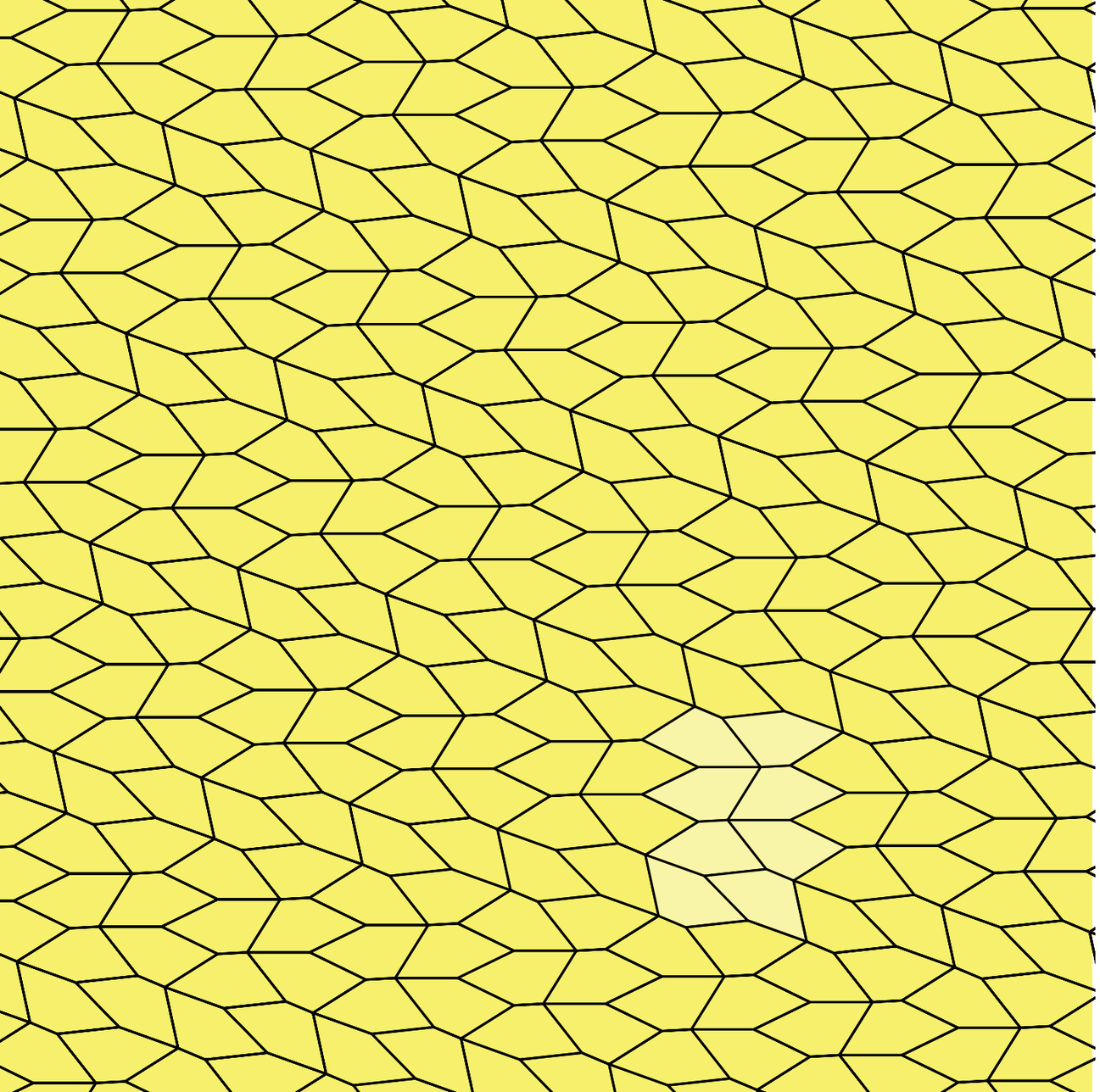


1968 / KERSHNER



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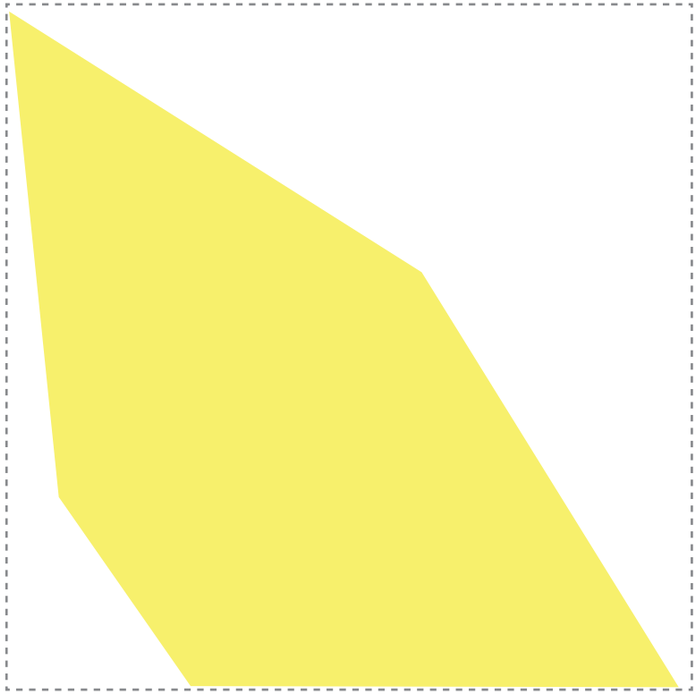
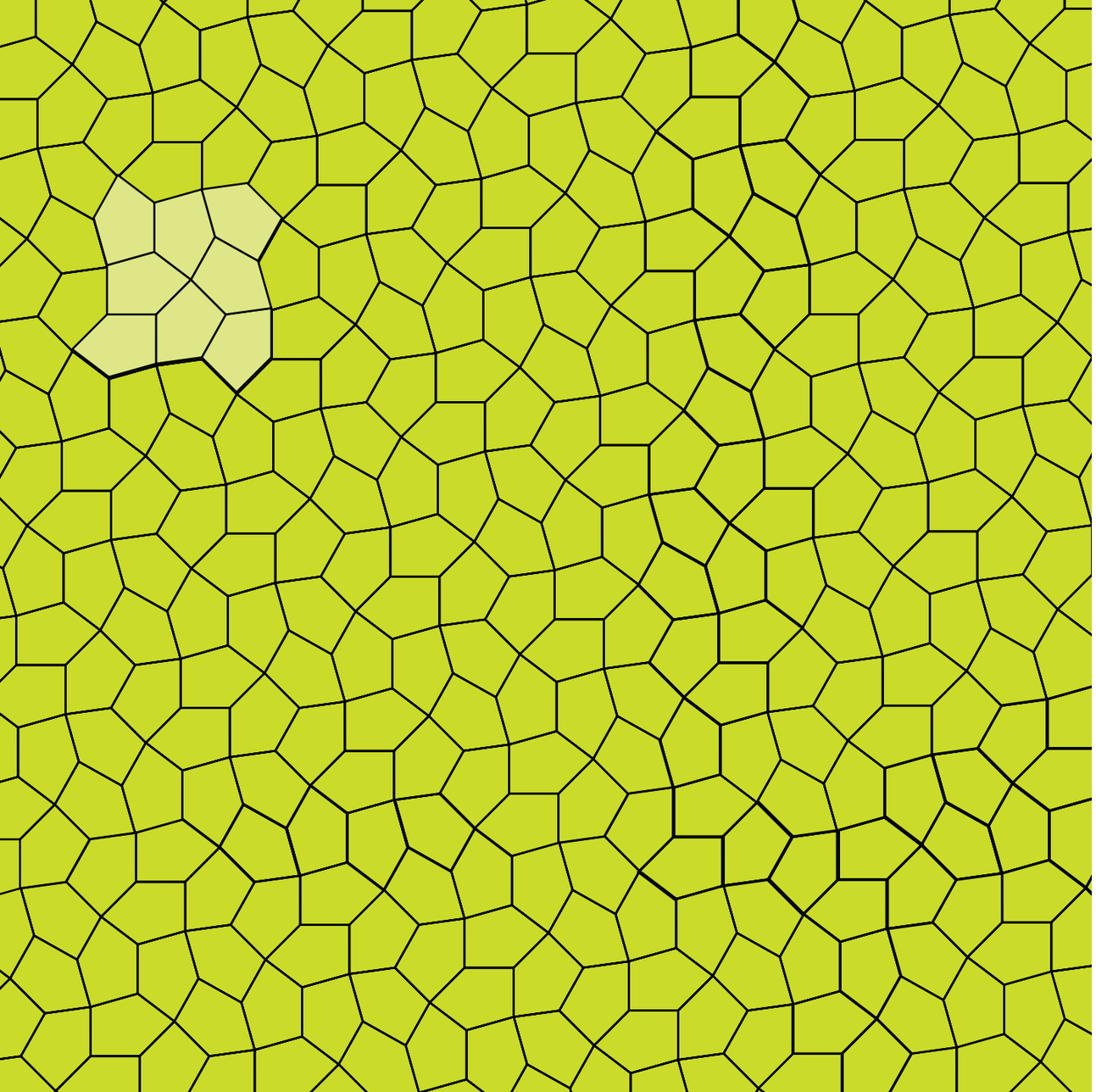


1968 / KERSHNER



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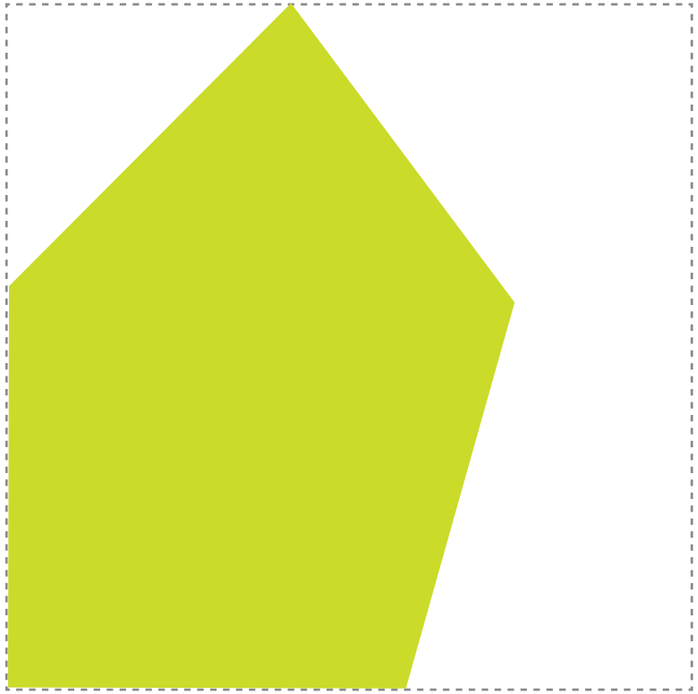
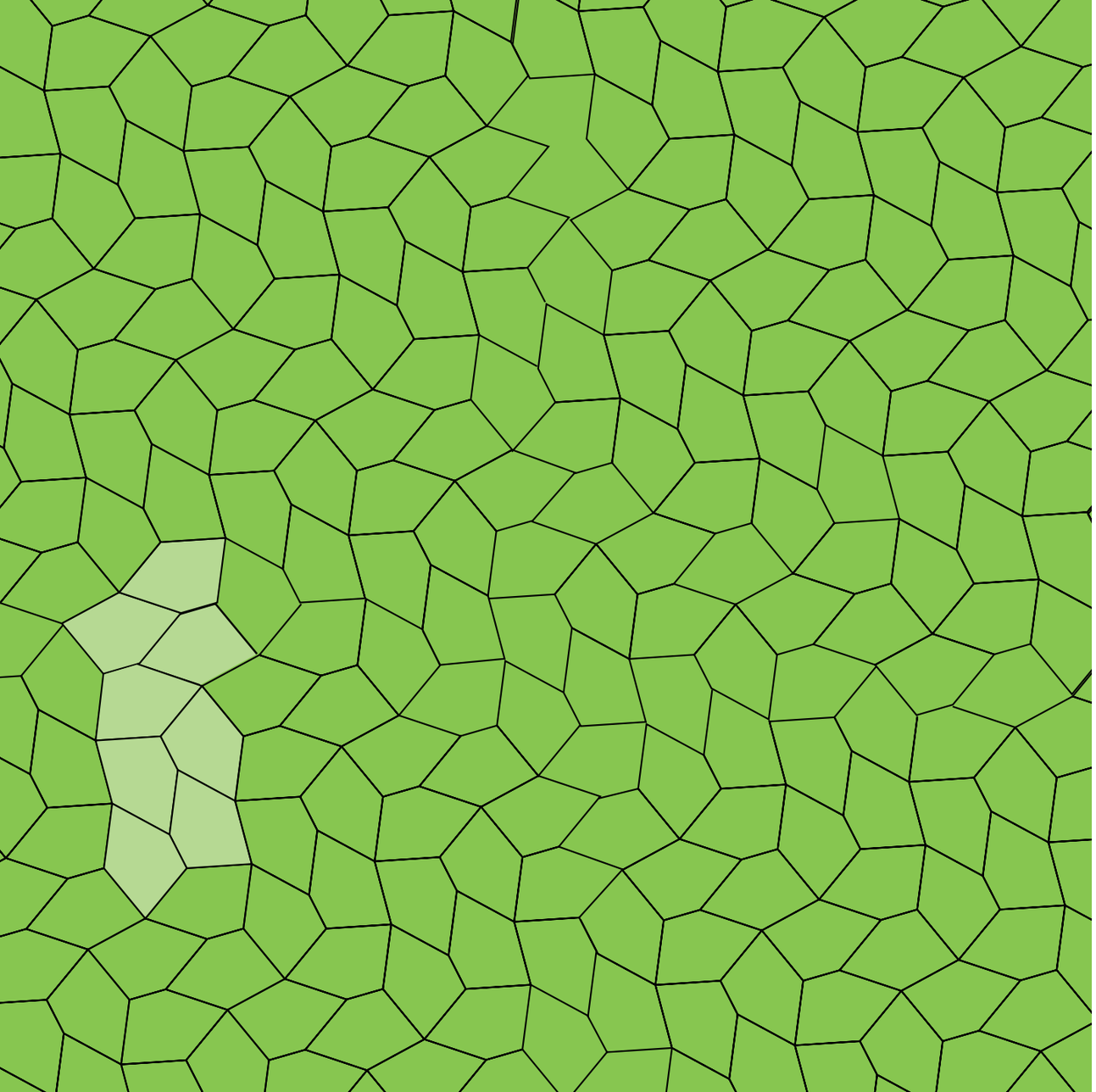


1968 / KERSHNER



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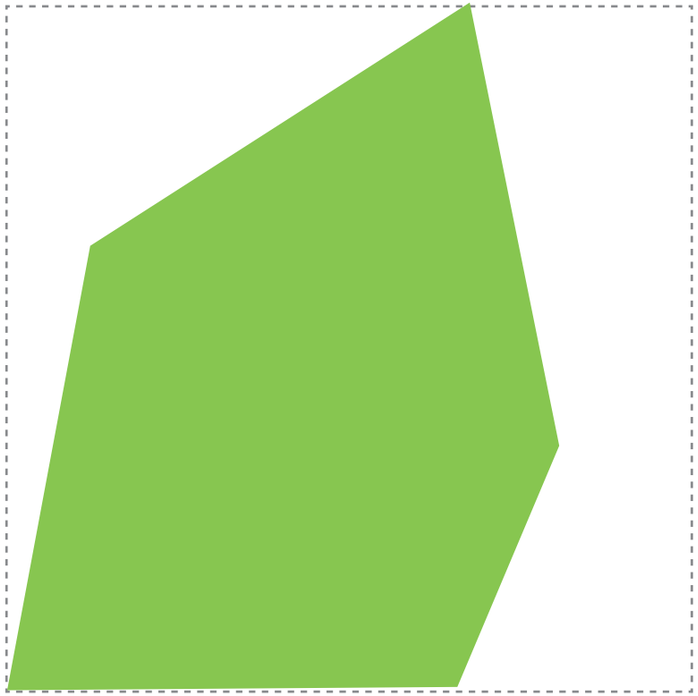
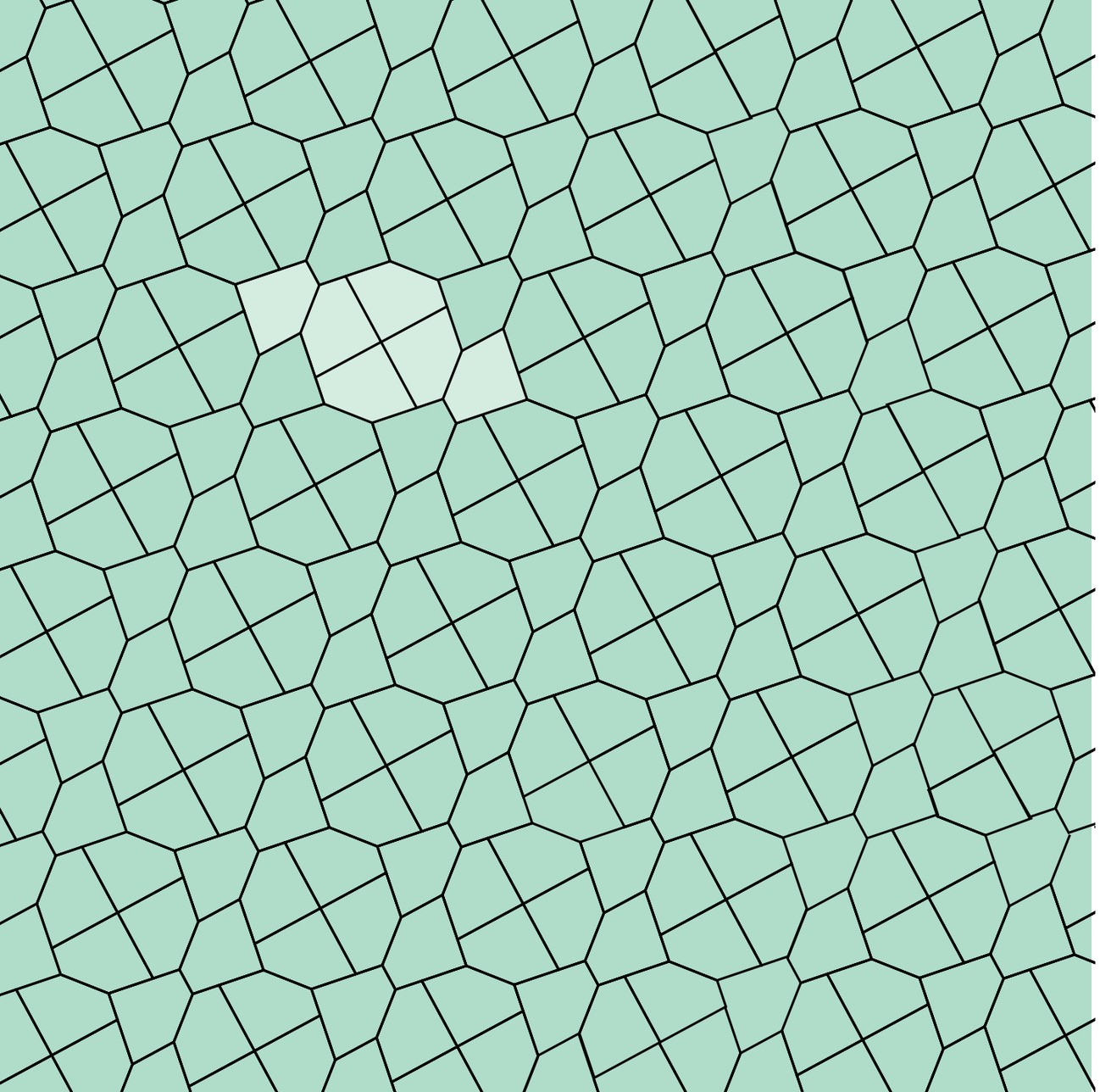


1977 / RICE



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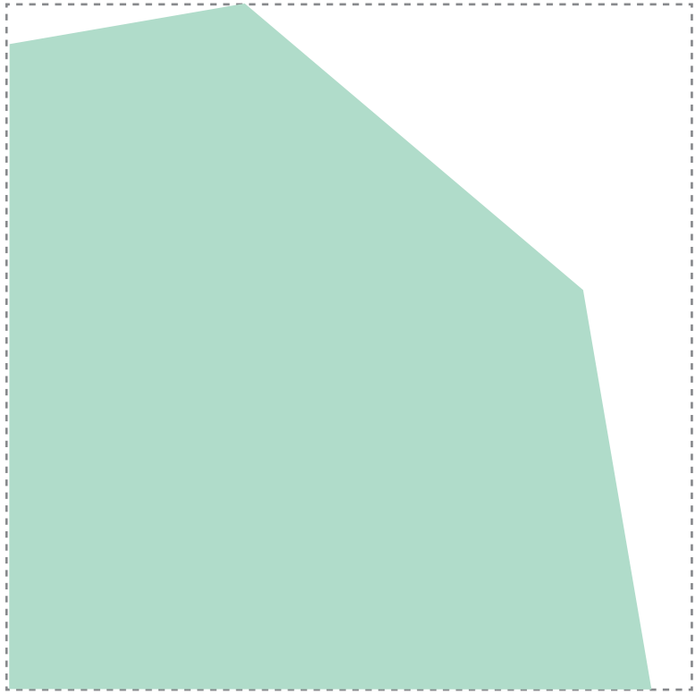
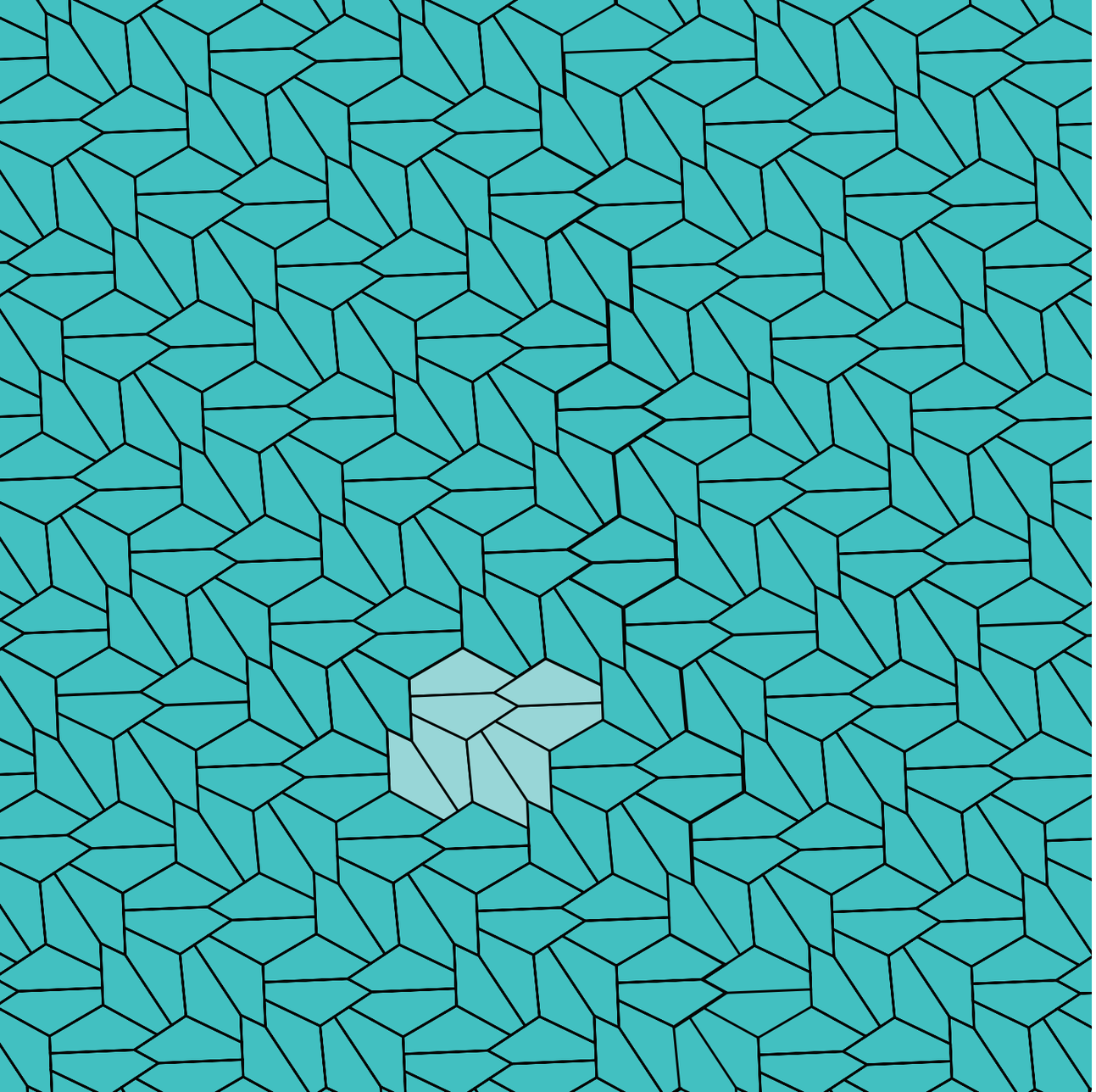


1975 / JAMES



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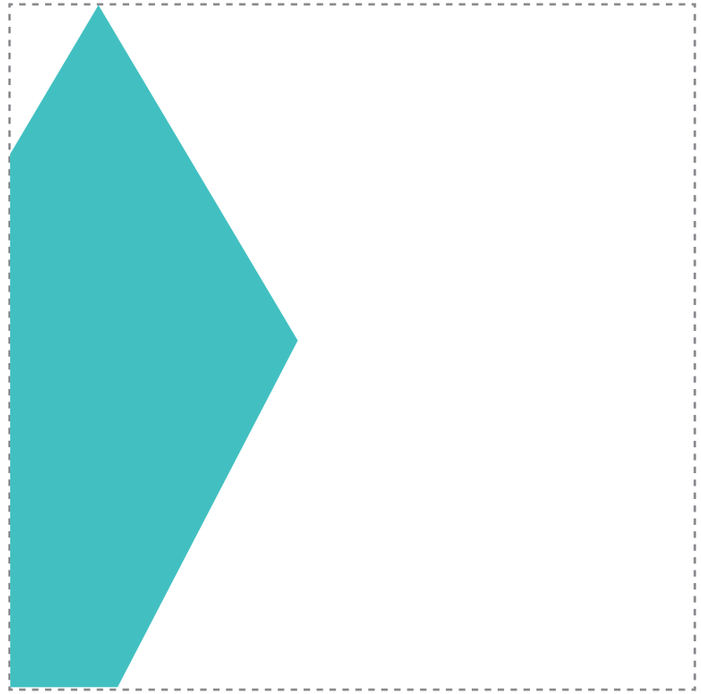
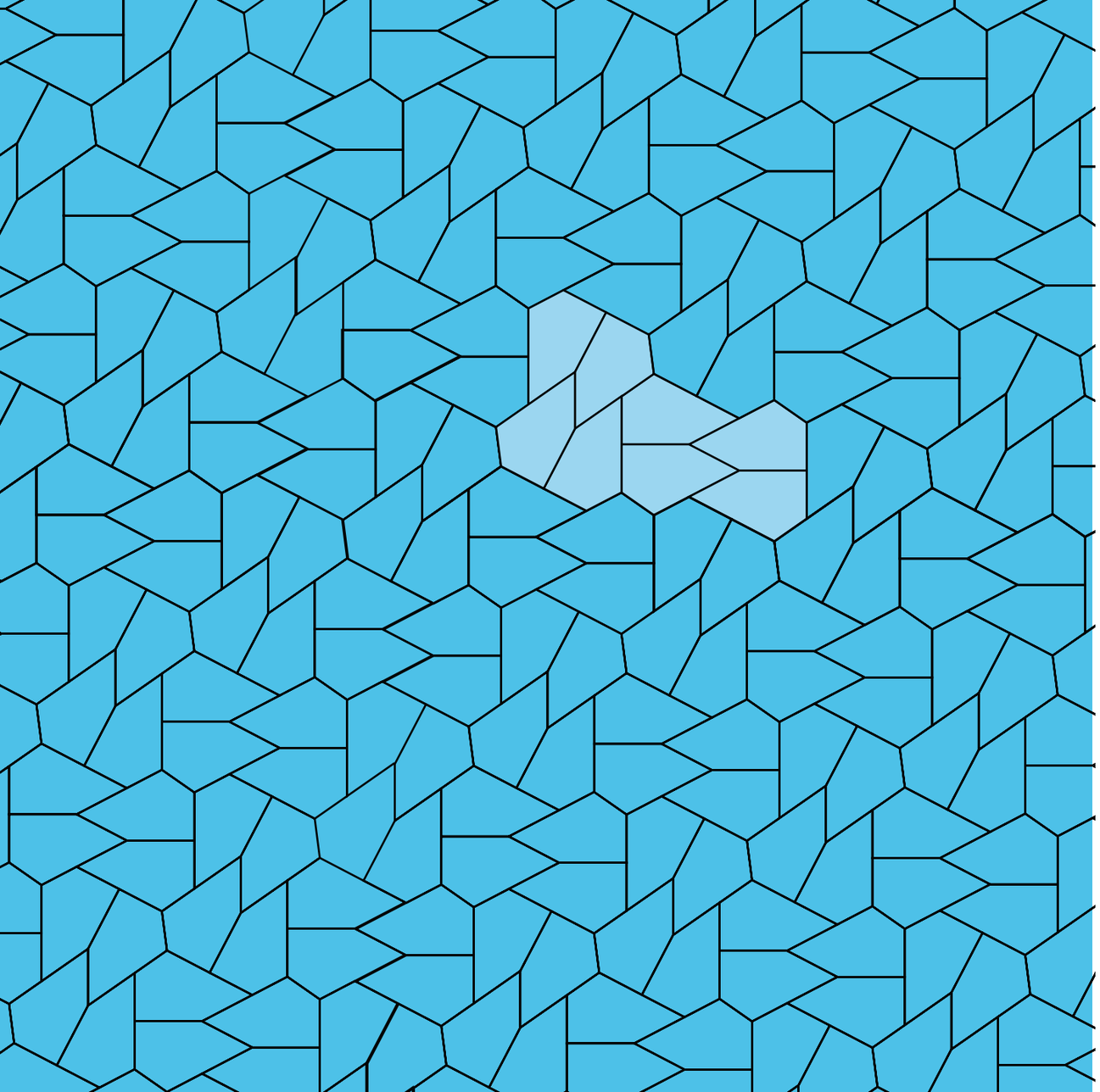


1977 / RICE



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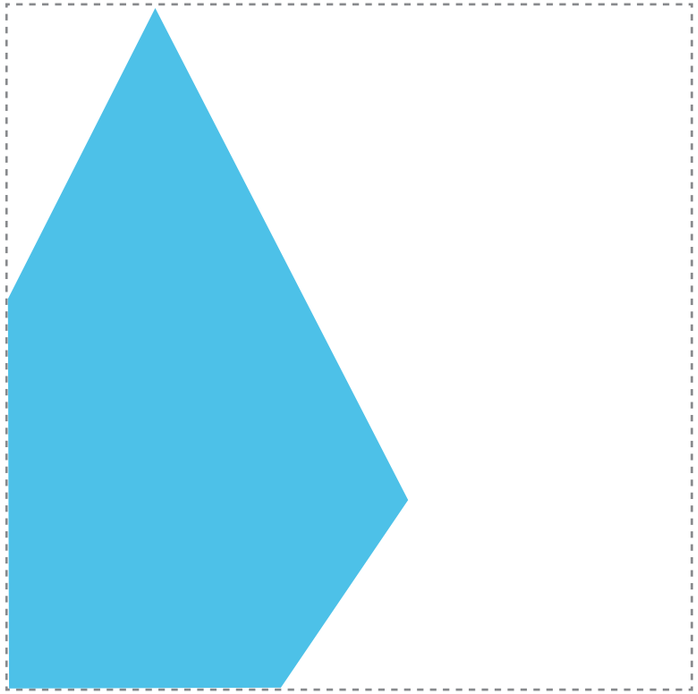

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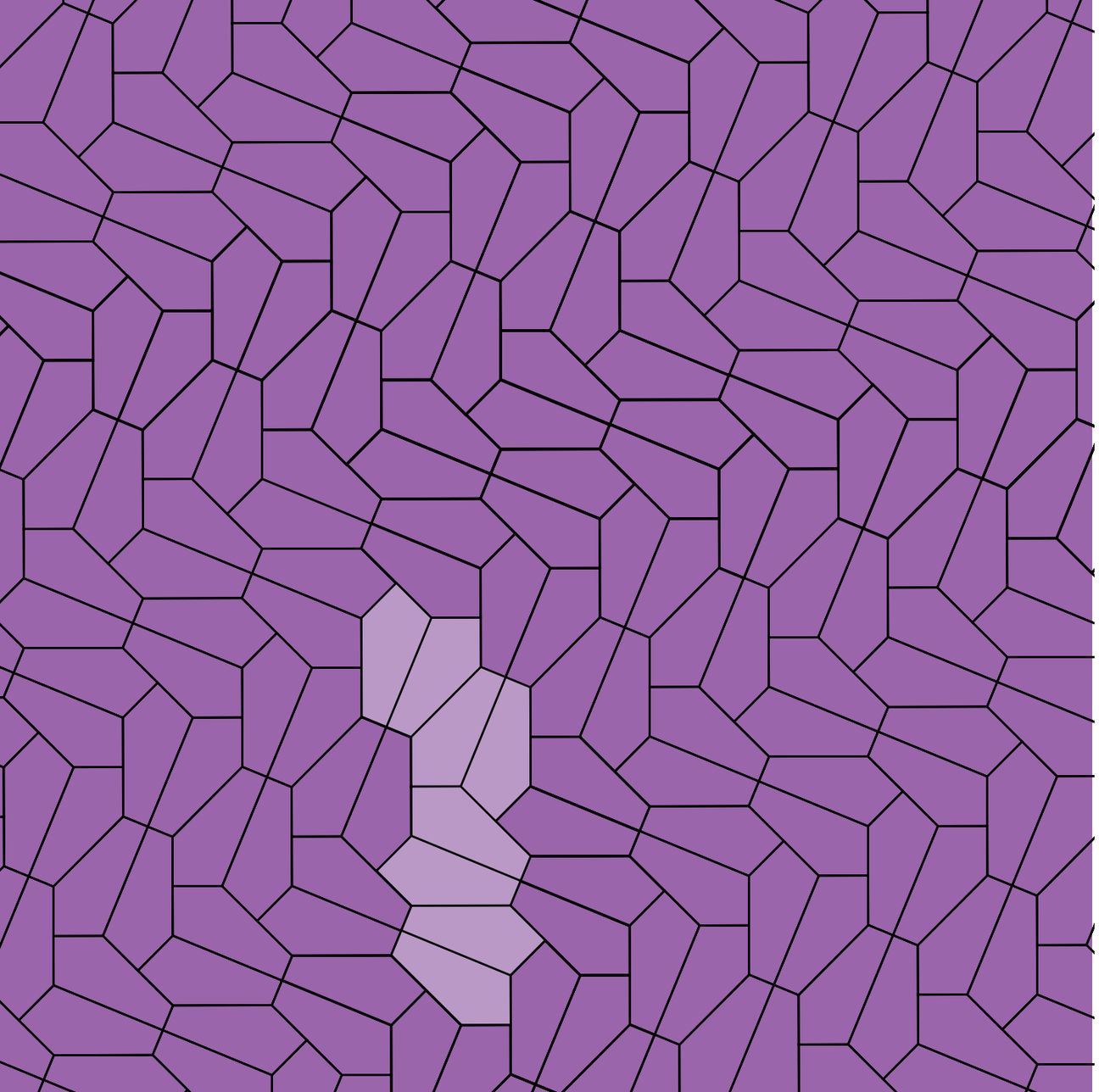

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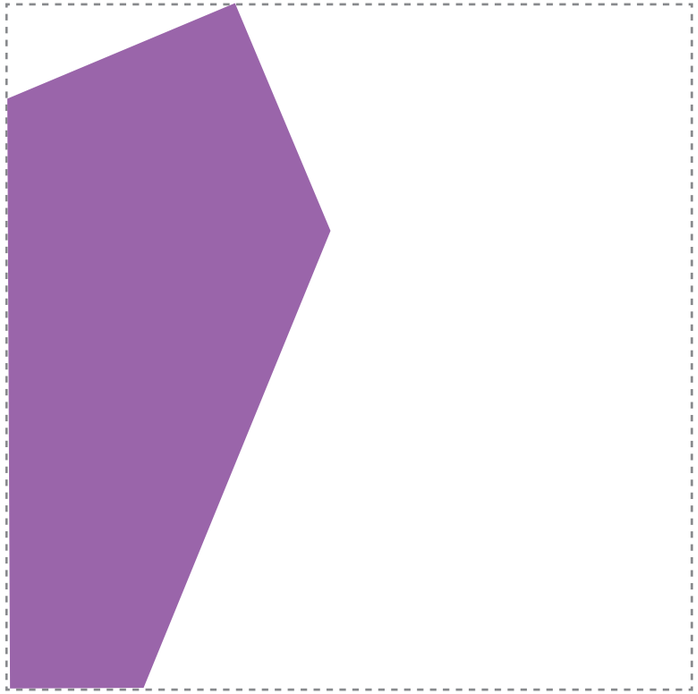
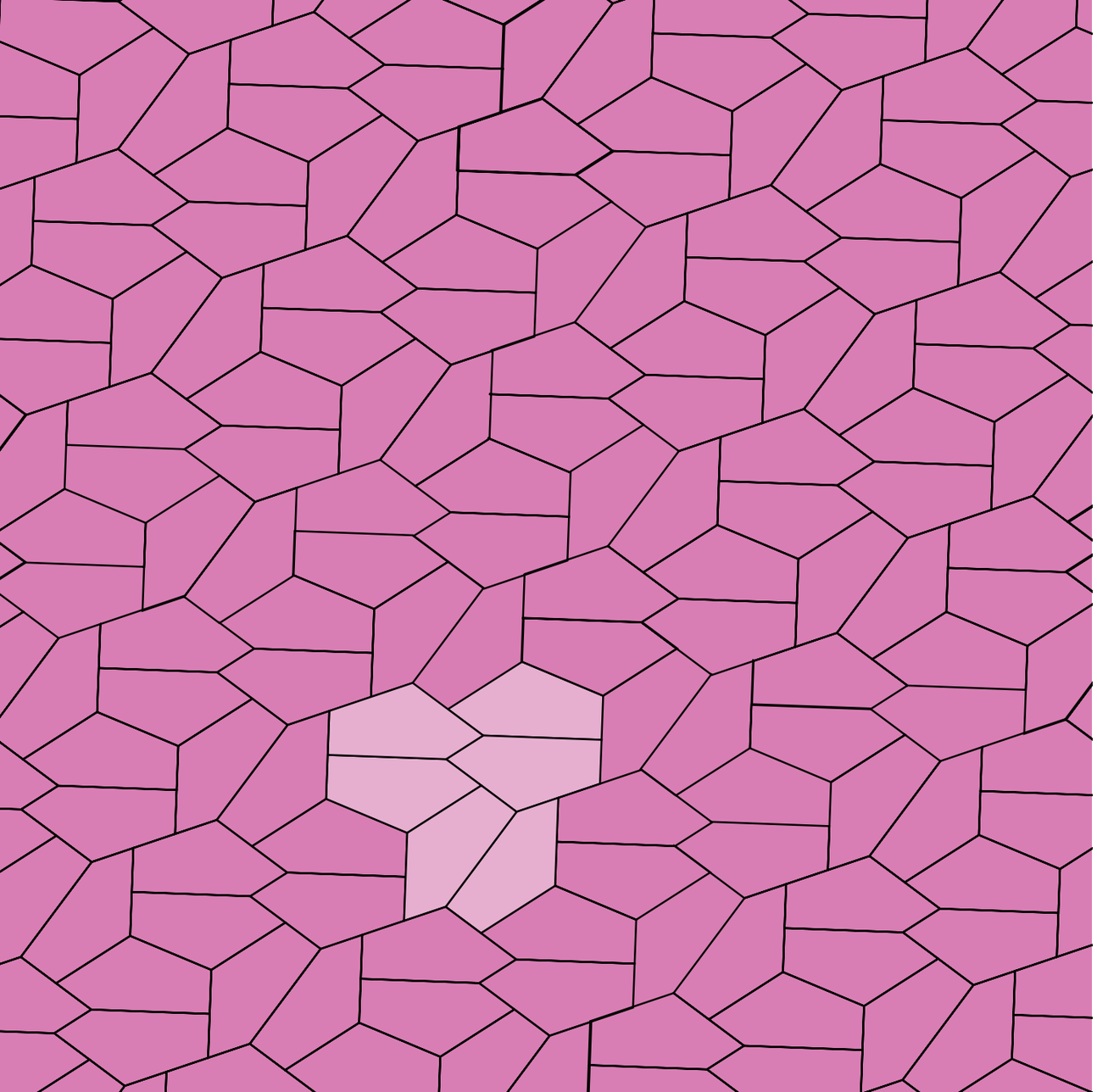


1977 / RICE



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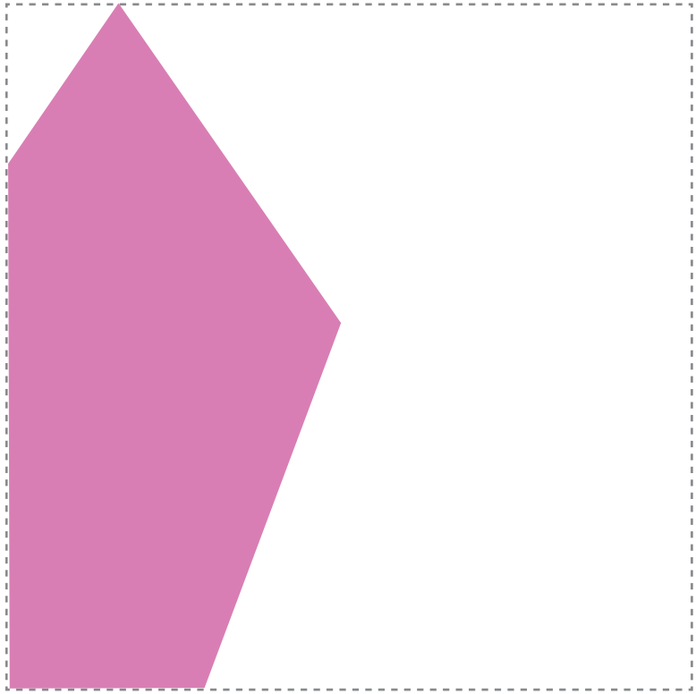
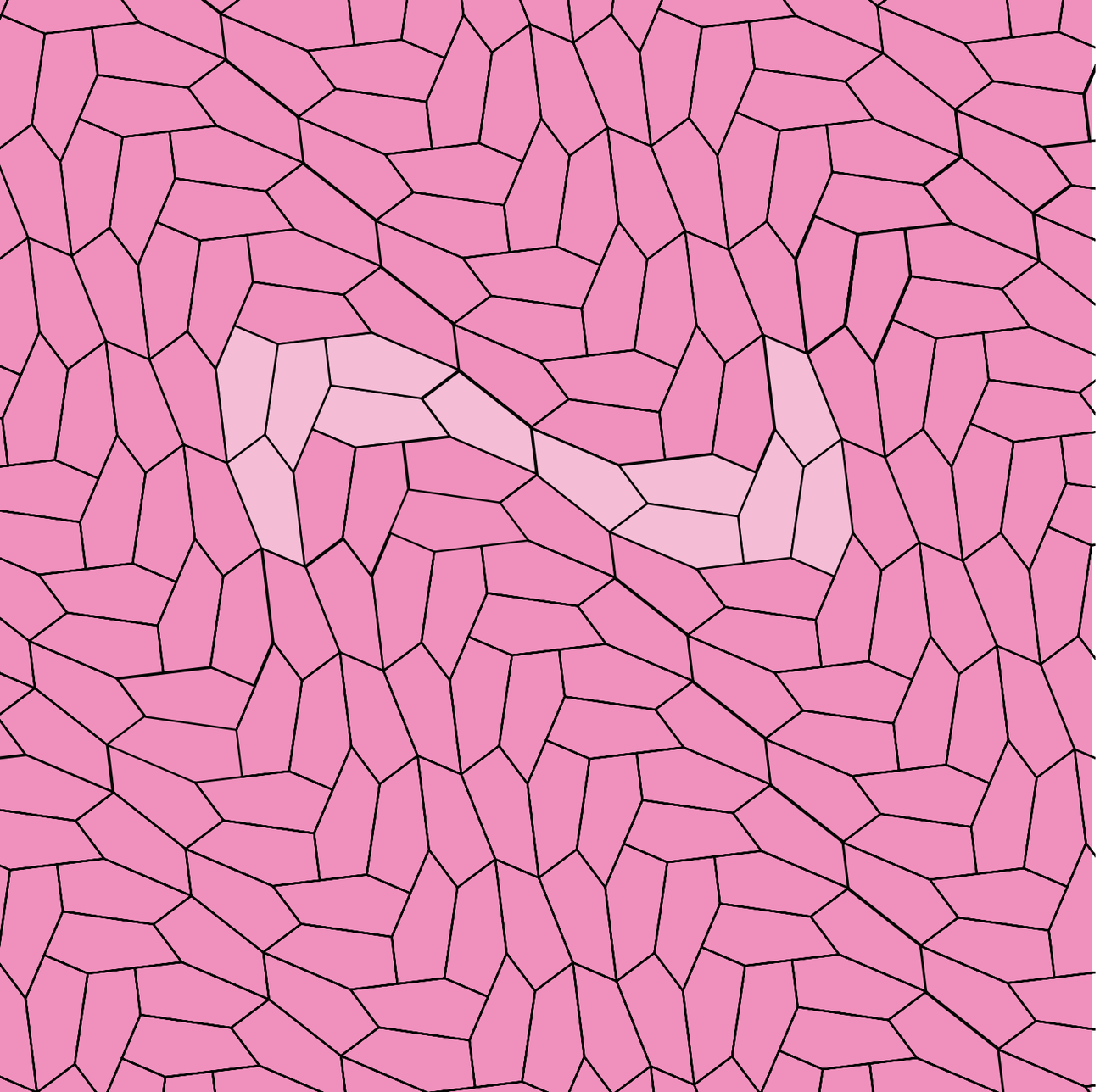


1985 / STEIN



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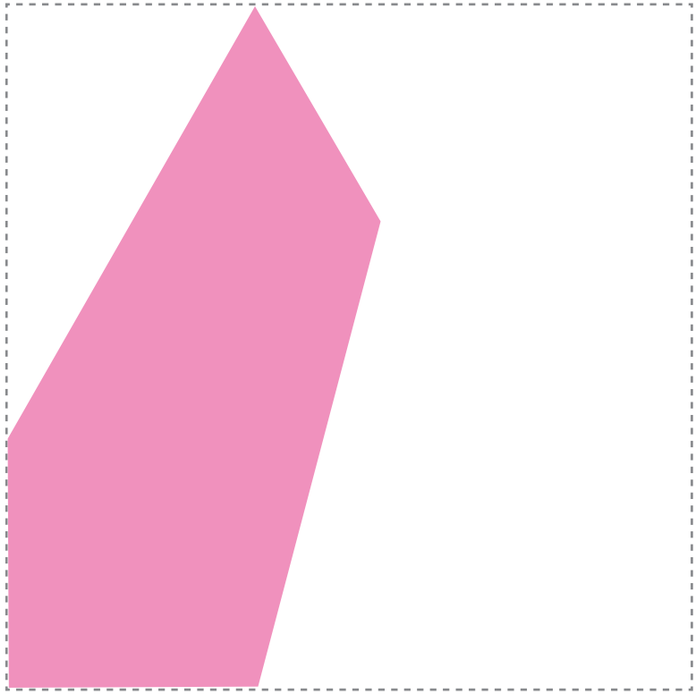
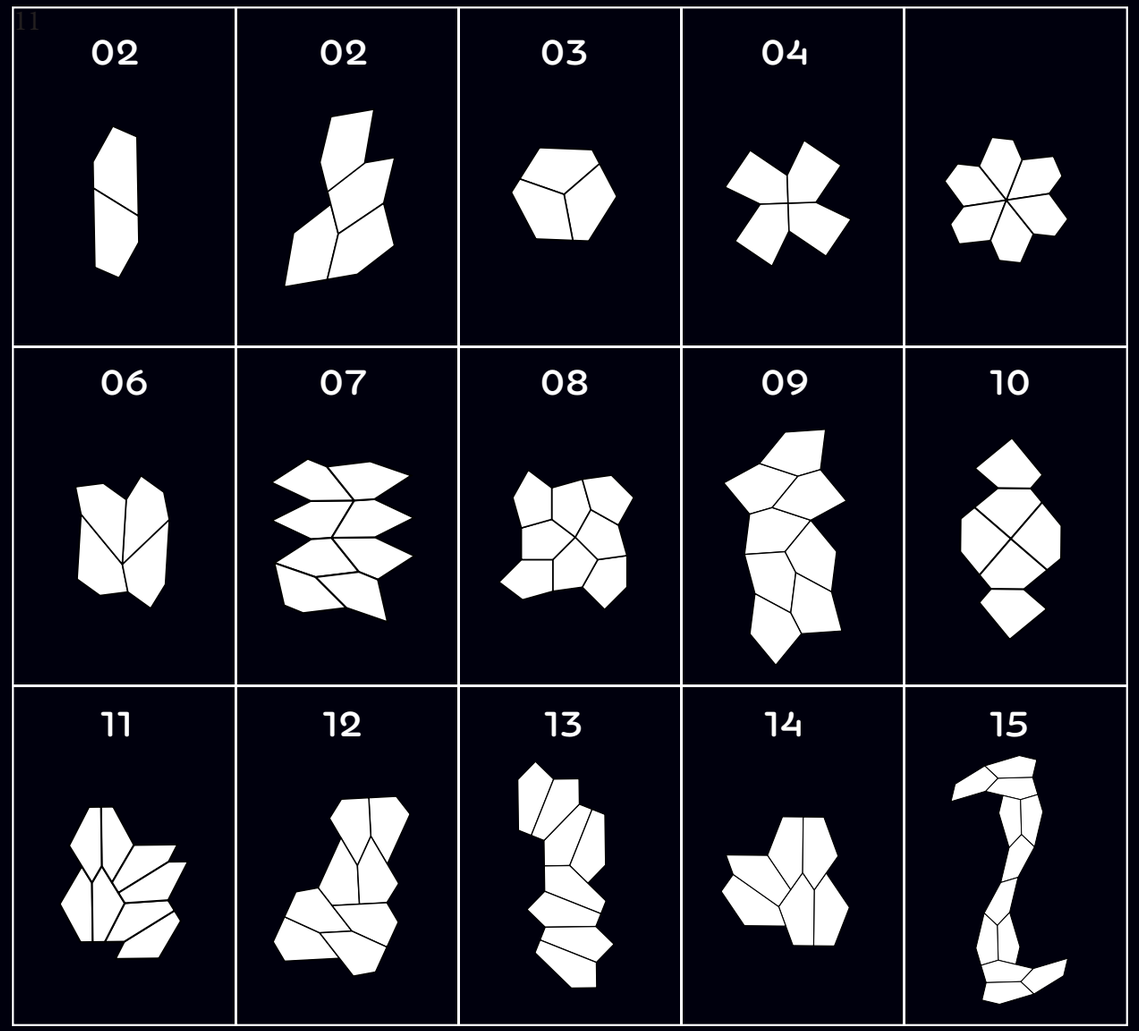


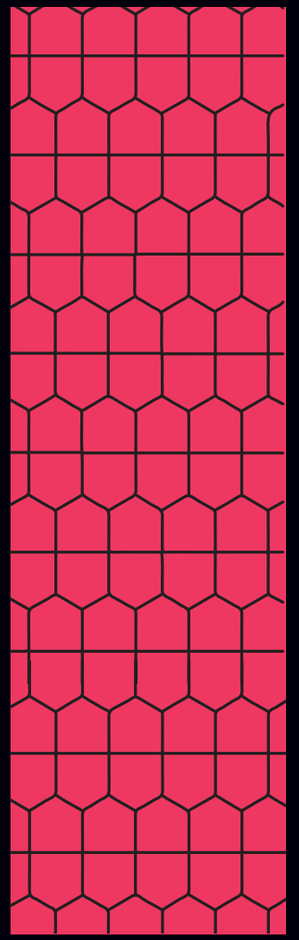
2015 / MCLOUD-MANN, MANN, VON DERAU

PRIMITIVE UNITS

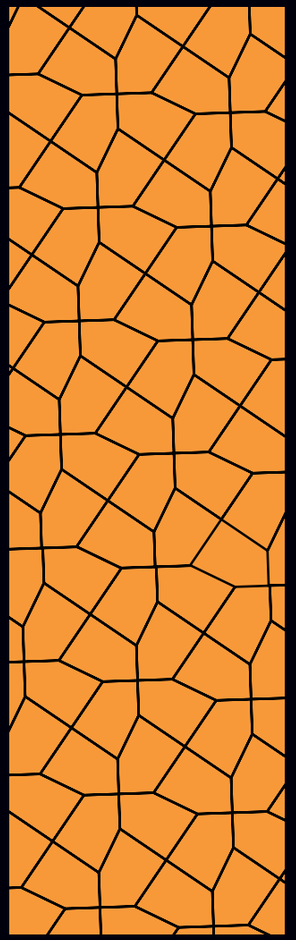


SOME PATTERNS ARE NAMED

PRISMATIC



CAIRO



FLORET