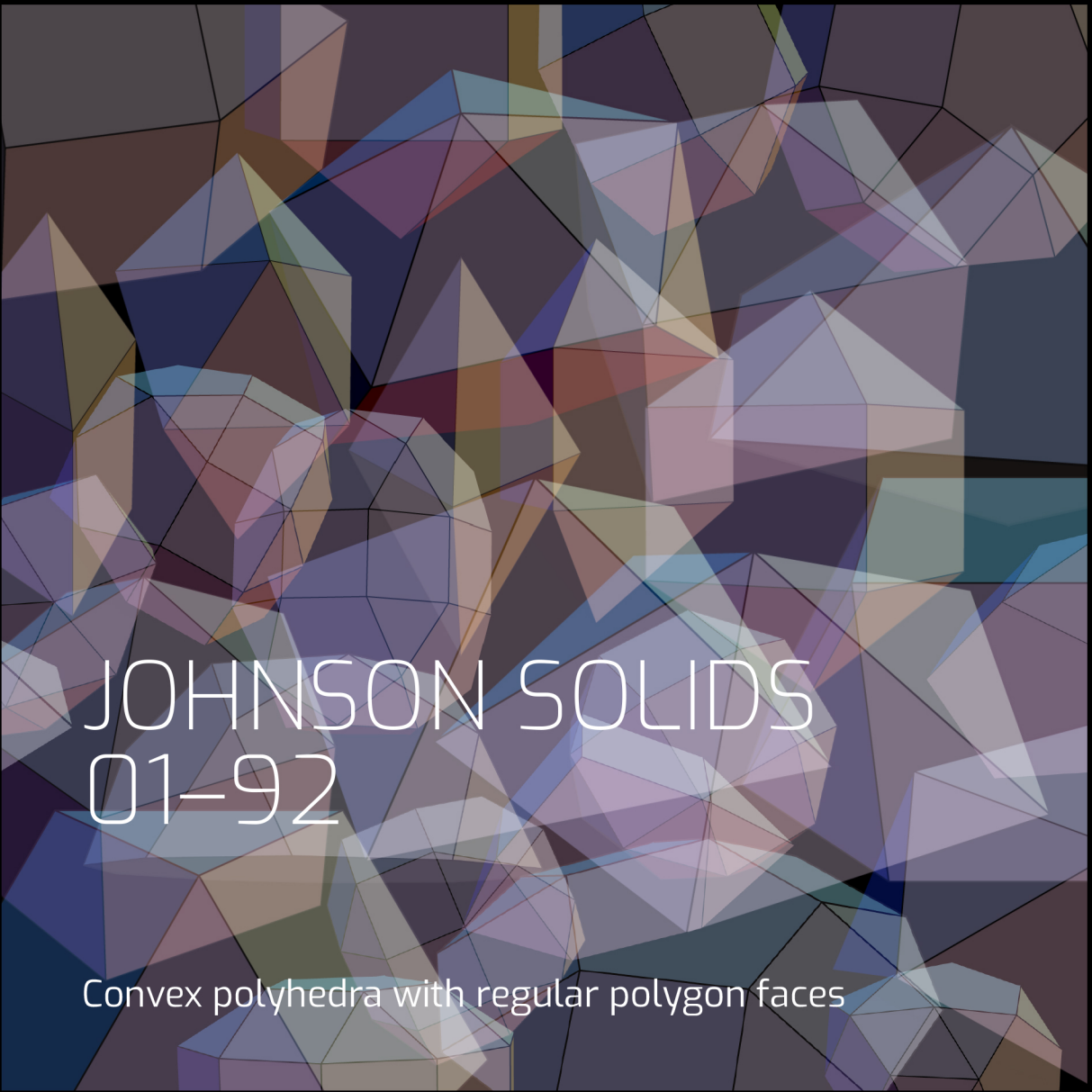

JOHNSON SOLIDS
01–92
Convex polyhedra with regular polygon faces

THE NINETY-TWO
JOHNSON SOLIDS

pointlineplane.xyz

Norman Johnson was born in Chicago in 1930.

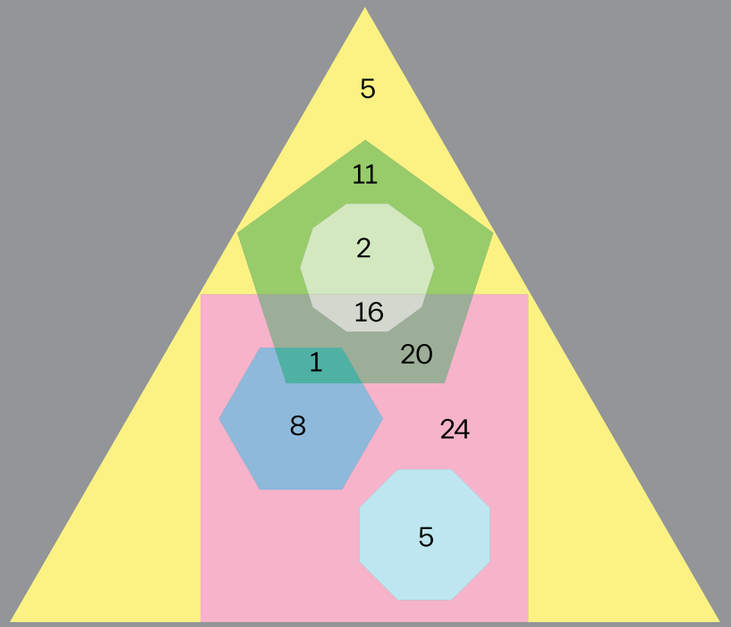
Johnson was a mathematician. In 1966, he identified 92 specific non-uniform convex solids with regular faces and speculated that no more could exist. After earning his PhD from the University of Toronto in that year, Johnson went on to hold a position in the Mathematics Department of Wheaton College in Massachusetts until his retirement in 1998. His PhD supervisor was the great mathematician H.S.M. Coxeter, known for his 1973 book *Regular Polytopes*, which enumerates the properties of perfect solids through higher dimensions.

In 1969, it was proven by mathematician Victor Zalgaller that Johnson was correct. There exist only 92 possible solids. They became known as the "Johnson Solids."

Just before his death, Johnson completed the book *Geometries and Transformations*, which was published by Cambridge University Press in 2018.

Norman Johnson died on July 13, 2017.

The 92 Johnson solids are convex polyhedra whose faces are a combination from six different regular polygon shapes. It turns out that only these shapes fit together in a certain number of ways to produce the polyhedra. The polygon faces must be “perfect,” meaning they have symmetry and same-sized sides: an equilateral triangle, a perfect square, and so on. Seven-sided polygons, for example, will not work. There are no Johnson solids composed of all six of the shapes. There are five Johnson solids composed of only triangles. This Venn diagram maps the number of shape combinations that comprise the 92 polyhedra. In the following pages, you will find the Johnson solids organized into groups according to their shapes. On the facing page is a summary.



NUMBER OF SIDES

- 3 TRIANGLE
- 4 SQUARE
- 5 PENTAGON
- 6 HEXAGON
- 8 OCTAGON
- 10 DECAGON

NUMBER OF JOHNSON SOLIDS

- △ = 5
- △ + □ = 24
- △ + ⬠ = 11
- △ + □ + ⬠ = 20
- △ + □ + ⬡ = 8
- △ + □ + ⯃ = 5
- △ + ⬠ + ⯃ = 2
- △ + □ + ⬠ + ⬡ = 1
- △ + □ + ⬠ + ⯃ = 16

TOTAL = 92

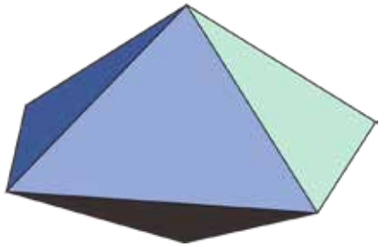
square pyramid § pentagonal pyramid § triangular cupola § square cupola § pentagonal cupola § pentagonal rotunda § elongated triangular pyramid § elongated square pyramid § elongated pentagonal pyramid § gyroelongated square pyramid § gyroelongated pentagonal pyramid § triangular bipyramid § pentagonal bipyramid § elongated triangular bipyramid § elongated square bipyramid § elongated pentagonal bipyramid § gyroelongated square bipyramid § elongated triangular cupola § elongated square cupola § elongated pentagonal cupola § elongated pentagonal rotunda § gyroelongated triangular cupola § gyroelongated square cupola § gyroelongated pentagonal cupola § gyroelongated pentagonal rotunda § gyrobifastigium § triangular orthobicupola § square orthobicupola § square gyrobicupola § pentagonal orthobicupola § pentagonal gyrobicupola § pentagonal orthocupolarotunda § pentagonal gyrocupola rotunda § pentagonal orthobirotunda § elongated triangular orthobicupola § elongated triangular gyrobicupola § elongated square gyrobicupola § elongated pentagonal orthobicupola § elongated pentagonal gyrobicupola § elongated pentagonal orthocupolarotunda § elongated pentagonal gyrocupolarotunda § elongated pentagonal orthobirotunda § elongated pentagonal gyrobirotunda § gyroelongated triangular bicupola § gyroelongated square bicupola § gyroelongated pentagonal bicupola § gyroelongated pentagonal cupolarotunda § gyroelongated pentagonal birotunda § augmented triangular prism § biaugmented triangular prism § triaugmented triangular prism § augmented pentagonal prism § biaugmented pentagonal prism § augmented hexagonal prism § parabiaugmented hexagonal prism § metabiaugmented hexagonal prism § triaugmented hexagonal prism § augmented dodecahedron § parabiaugmented dodecahedron § metabiaugmented dodecahedron § triaugmented dodecahedron § metabidiminished icosahedron § tridiminished icosahedron § augmented tridiminished icosahedron § augmented truncated tetrahedron § augmented truncated cube § biaugmented truncated cube § augmented truncated dodecahedron § parabiaugmented truncated dodecahedron § metabiaugmented truncated dodecahedron § triaugmented truncated dodecahedron § gyrate rhombicosidodecahedron § parabigyrate rhombicosidodecahedron § metabigyrate rhombicosidodecahedron § trigyrate rhombicosidodecahedron § diminished rhombicosidodecahedron § paragyrate diminished rhombicosidodecahedron § metagyrate diminished rhombicosidodecahedron § bigyrate diminished rhombicosidodecahedron § parabidiminished rhombicosidodecahedron § metabidiminished rhombicosidodecahedron § gyrate bidiminished rhombicosidodecahedron § tridiminished rhombicosidodecahedron § snub disphenoid § snub square antiprism § sphenocorona § augmented sphenocorona § sphenomegacorona § hebesphenomegacorona § disphenocingulum § bilunabirotunda § triangular hebesphenorotunda

THE SOLIDS

Triangle-only faced

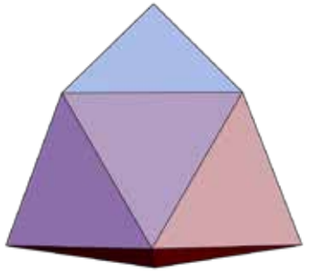
Five solids are called *deltahedra*, meaning they are made of only triangles. Johnson solid triangles are always equilateral.

J12
TRIANGULAR BIPYRAMID

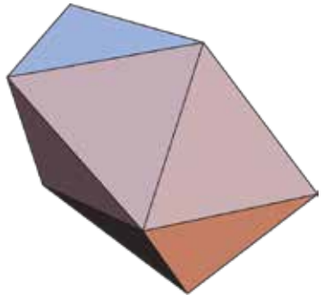


J13
PENTAGONAL BIPYRAMID

J17
GYROELONGATED SQUARE BIPYRAMID

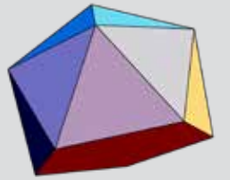


J51
TRIAUGMENTED TRIANGULAR PRISM

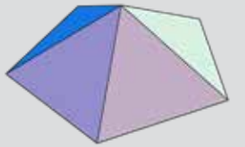


Triangle + Pentagon faced

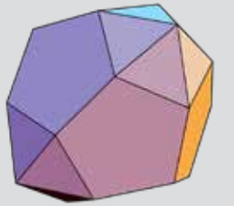
Eleven solids have only triangle and pentagon faces.



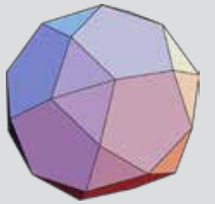
J2
PENTAGONAL PYRAMID



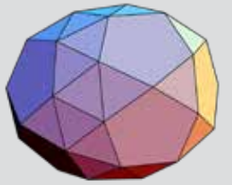
J11
GYROELONGATED PENTAGONAL PYRAMID



J60
METABIAUGMENTED DODECAHEDRON

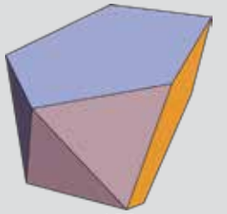


J34
PENTAGONAL ORTHOBIROTUNDA

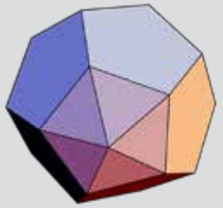


J48
GYROELONGATED PENTAGONAL ORTHOBIROTUNDA

J61
TRIAUGMENTED DODECAHEDRON



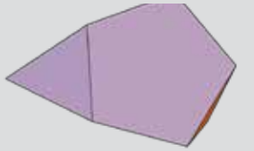
J62
METABIDIMINISHED ICOSAHEDRON



J58
AUGMENTED DODECAHEDRON

J59
PARABIAUGMENTED DODECAHEDRON

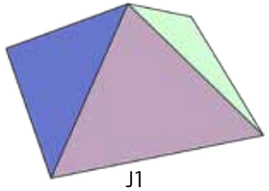
J63
TRIDIMINISHED ICOSAHEDRON



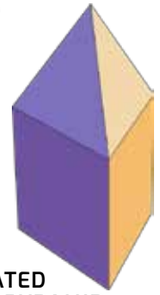
J64
AUGMENTED TRIDIMINISHED ICOSAHEDRON

Triangle + Square faced

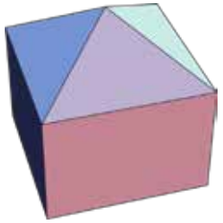
Twenty-four solids have faces made up of only triangles and squares.



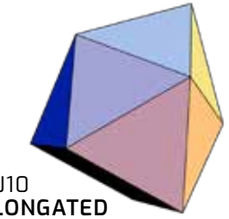
J1
SQUARE PYRAMID



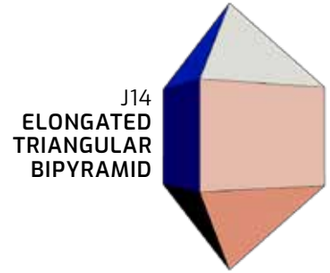
J7
ELONGATED
TRIANGULAR PYRAMID



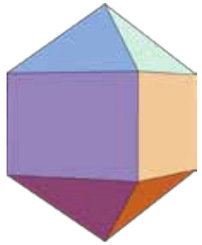
J8
ELONGATED
SQUARE
PYRAMID



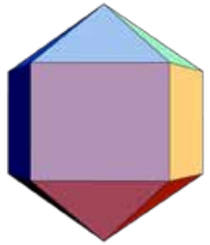
J10
GYROELONGATED
SQUARE PYRAMID



J14
ELONGATED
TRIANGULAR
BIPYRAMID

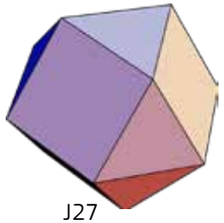


J15
ELONGATED
SQUARE
BIPYRAMID

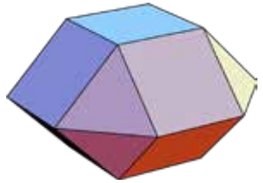


J16
ELONGATED
PENTAGONAL
BIPYRAMID

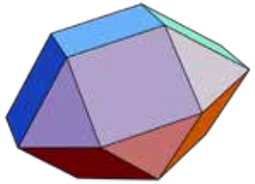
J26
GYROBIFASTIGIUM



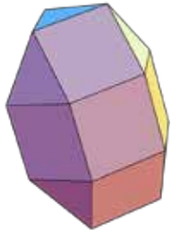
J27
TRIANGULAR
ORTHOBICUPOLA



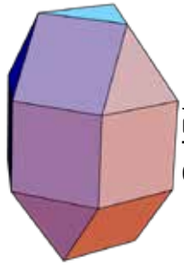
J28
SQUARE
ORTHOBICUPOLA



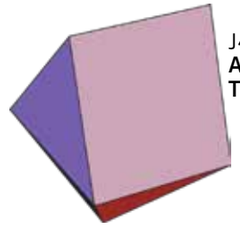
J29
SQUARE
GYROBICUPOLA



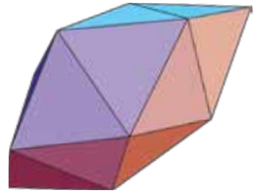
J35
ELONGATED
TRIANGULAR
ORTHOBICUPOLA



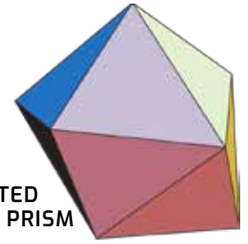
J36
ELONGATED
TRIANGULAR
GYROBICUPOLA



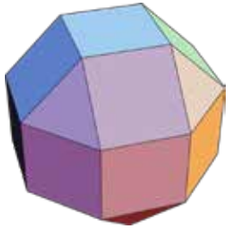
J49
AUGMENTED
TRIANGULAR PRISM



J88
SPHENOMEGACORONA

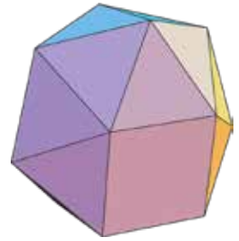


J50
BIUGMENTED
TRIANGULAR PRISM

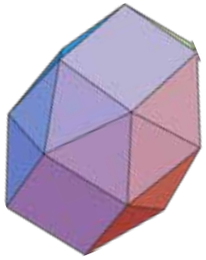


J37
ELONGATED
SQUARE
GYROBICUPOLA

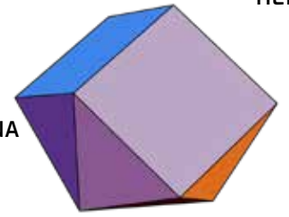
J85
SNUB SQUARE
ANTIPRISM



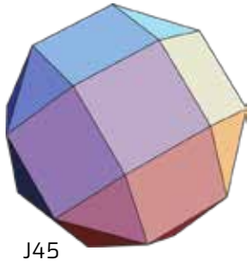
J89
HEBESPHENOMEGACORONA



J44
GYROELONGATED
TRIANGULAR
BICUPOLA

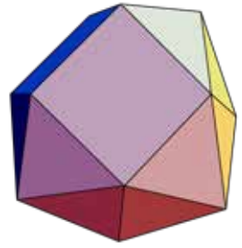


J86
SPHENOCORONA



J45
GYROELONGATED
SQUARE BICUPOLA

J87
AUGMENTED
SPHENOCORONA

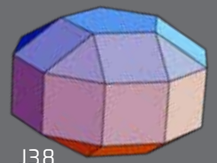


J90
DISPHENOCINGULUM

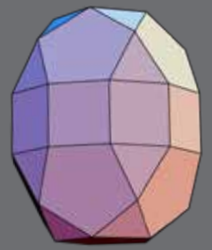
Triangle + Square + Pentagon faced

Twenty solids have only triangle, square and pentagon faces

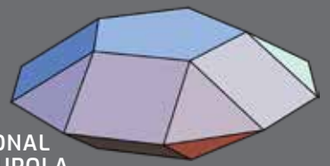
J9
ELONGATED
PENTAGONAL
PYRAMID



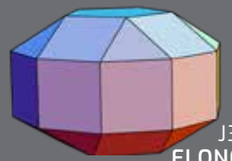
J38
ELONGATED
PENTAGONAL
ORTHOBICUPOLA



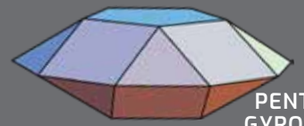
J42
ELONGATED
PENTAGONAL
ORTHOBIROTUNDA



J30
PENTAGONAL
ORTHOBICUPOLA

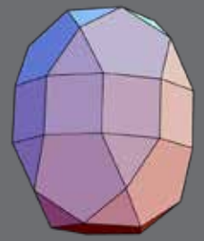


J39
ELONGATED
PENTAGONAL
GYROBICUPOLA

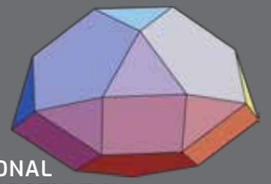


J31
PENTAGONAL
GYROBICUPOLA

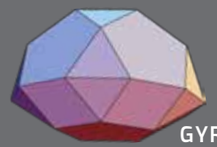
J40
ELONGATED
PENTAGONAL
ORTHOCUPOLAROTUNDA



J43
ELONGATED
PENTAGONAL
GYROBIROTUNDA

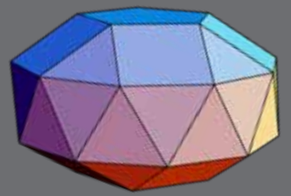


J32
PENTAGONAL
ORTHOCUPOLAROTUNDA



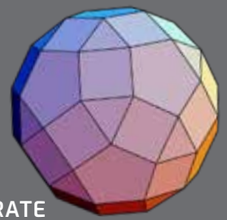
J33
PENTAGONAL
GYROCUPOLAROTUNDA

J41
ELONGATED
PENTAGONAL
GYROCUPOLAROTUNDA



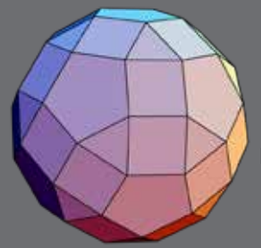
J46
GYROELONGATED
PENTAGONAL
BICUPOLA

J72
GYRATE
RHOMBICOSIDODECAHEDRON



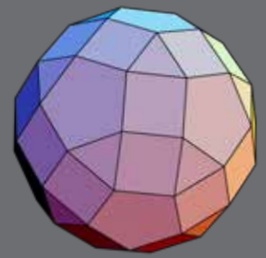
J73
PARABIGYRATE
RHOMBICOSIDODECAHEDRON

J47
GYROELONGATED
PENTAGONAL
CUPOLAROTUNDA

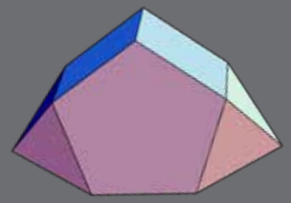


J74
METABIGYRATE
RHOMBICOSIDODECAHEDRON

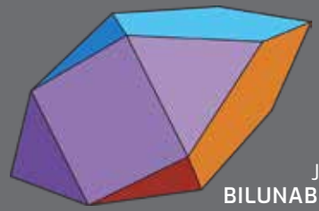
J52
AUGMENTED
PENTAGONAL
PRISM



J75
TRIGYRATE
RHOMBICOSIDODECAHEDRON



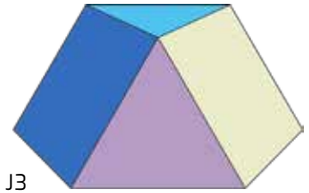
J53
BIAUGMENTED
PENTAGONAL
PRISM



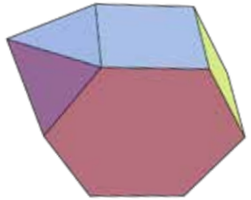
J91
BILUNABIROTUNDA

Triangle + Square + Hexagon faced

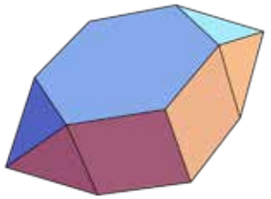
Eight solids have only triangle, square and hexagon faces



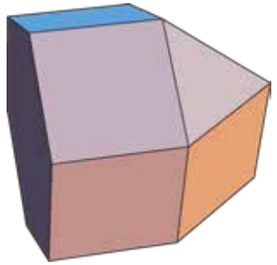
J3
TRIANGULAR
CUPOLA



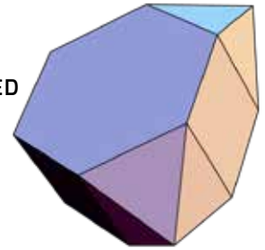
J54
AUGMENTED
HEXAGONAL
PRISM



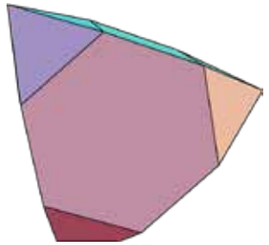
J55
PARABIAUGMENTED
HEXAGONAL
PRISM



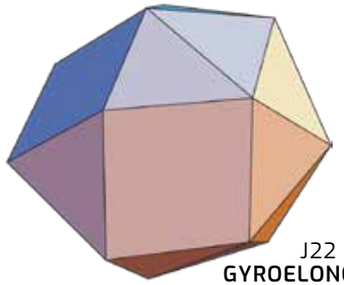
J18
ELONGATED
TRIANGULAR
CUPOLA



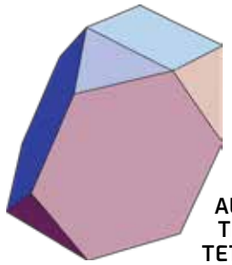
J56
METABIAUGMENTED
HEXAGONAL
PRISM



J57
TRIAUGMENTED
HEXAGONAL
PRISM



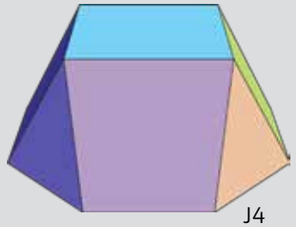
J22
GYROELONGATED
TRIANGULAR CUPOLA



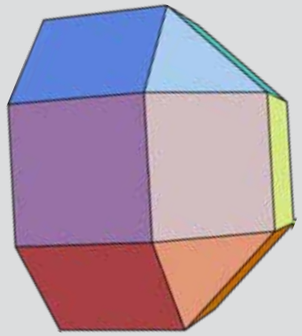
J65
AUGMENTED
TRUNCATED
TETRAHEDRON

Triangle + Square + Octagon faced

Five solids have only triangle, square and octagon faces

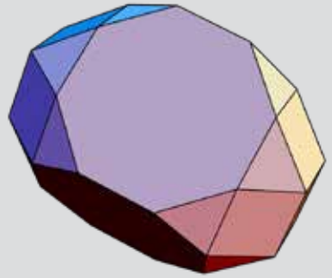


J4
SQUARE CUPOLA

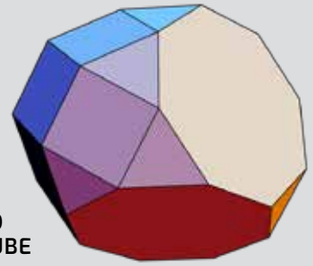


J19
ELONGATED
SQUARE CUPOLA

J23
GYROELONGATED
SQUARE CUPOLA



J67
BIAUGMENTED
TRUNCATED CUBE

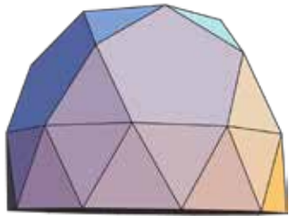


J66
AUGMENTED
TRUNCATED CUBE

Triangle + Pentagon + Decagon faced

Two solids have only triangle, pentagon and decagon faces

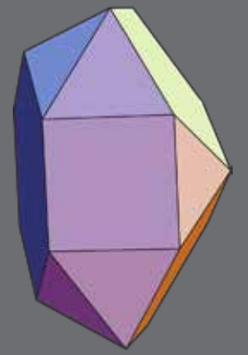
J6
PENTAGONAL ROTUNDA



J25
GYROELONGATED PENTAGONAL ROTUNDA

Triangle + Square + Pentagon + Hexagon faced

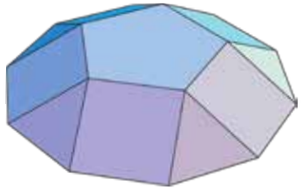
Only one solid has faced composed of triangle, square, pentagon and hexagon



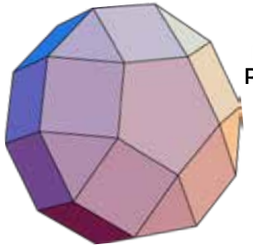
J92
TRIANGULAR HEBESPHENOROTUNDA

Triangle + Square + Pentagon + Decagon faced

Sixteen solids have only triangle, square, pentagon and decagon faces

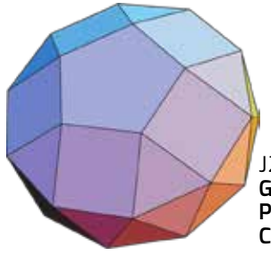


J5
PENTAGONAL CUPOLA

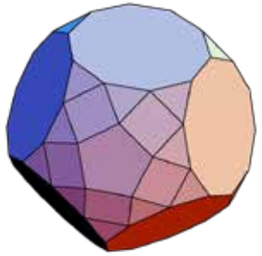


J20
ELONGATED PENTAGONAL CUPOLA

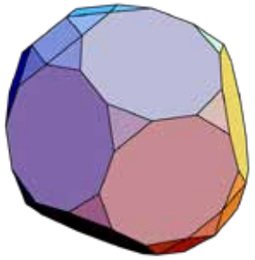
J21
ELONGATED PENTAGONAL ROTUNDA



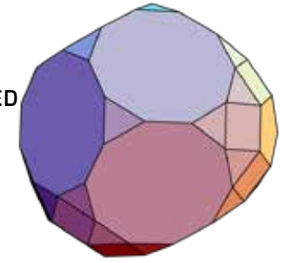
J24
GYROELONGATED PENTAGONAL CUPOLA



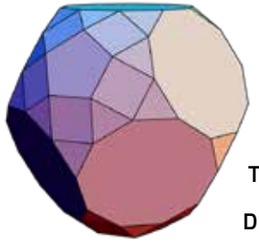
J68
AUGMENTED TRUNCATED DODECAHEDRON



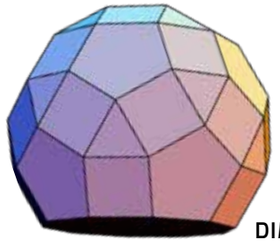
J69
PARABIAUGMENTED TRUNCATED DODECAHEDRON



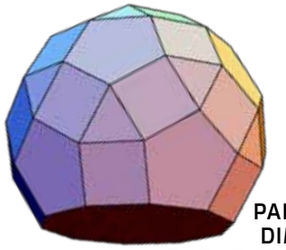
J70
METABIAUGMENTED TRUNCATED DODECAHEDRON



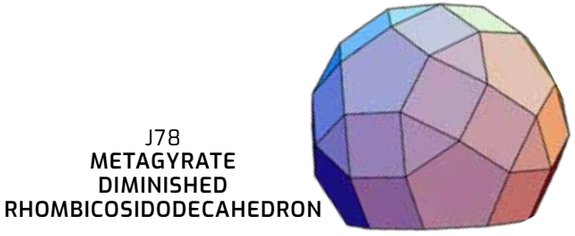
J71
TRIAUGMENTED TRUNCATED DODECAHEDRON



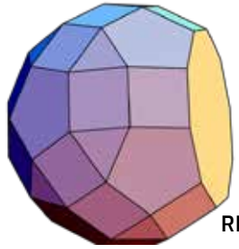
J76
DIMINISHED RHOMBICOSIDODECAHEDRON



J77
PARAGYRATE DIMINISHED RHOMBICOSIDODECAHEDRON

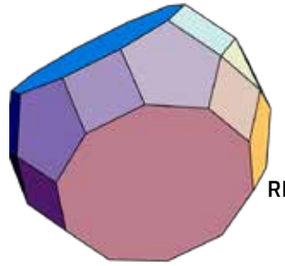


J78
METAGYRATE DIMINISHED RHOMBICOSIDODECAHEDRON

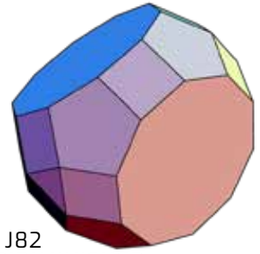


J79
BIGYRATE DIMINISHED RHOMBICOSIDODECAHEDRON

J80
PARABIDIMINISHED RHOMBICOSIDODECAHEDRON

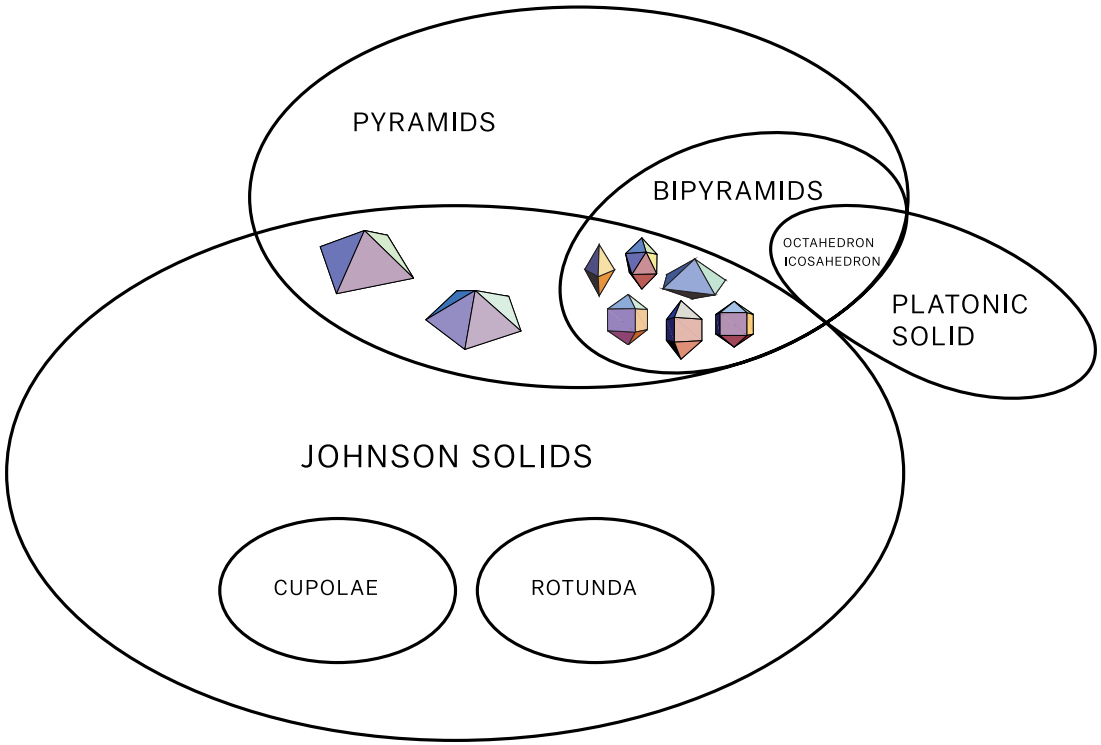


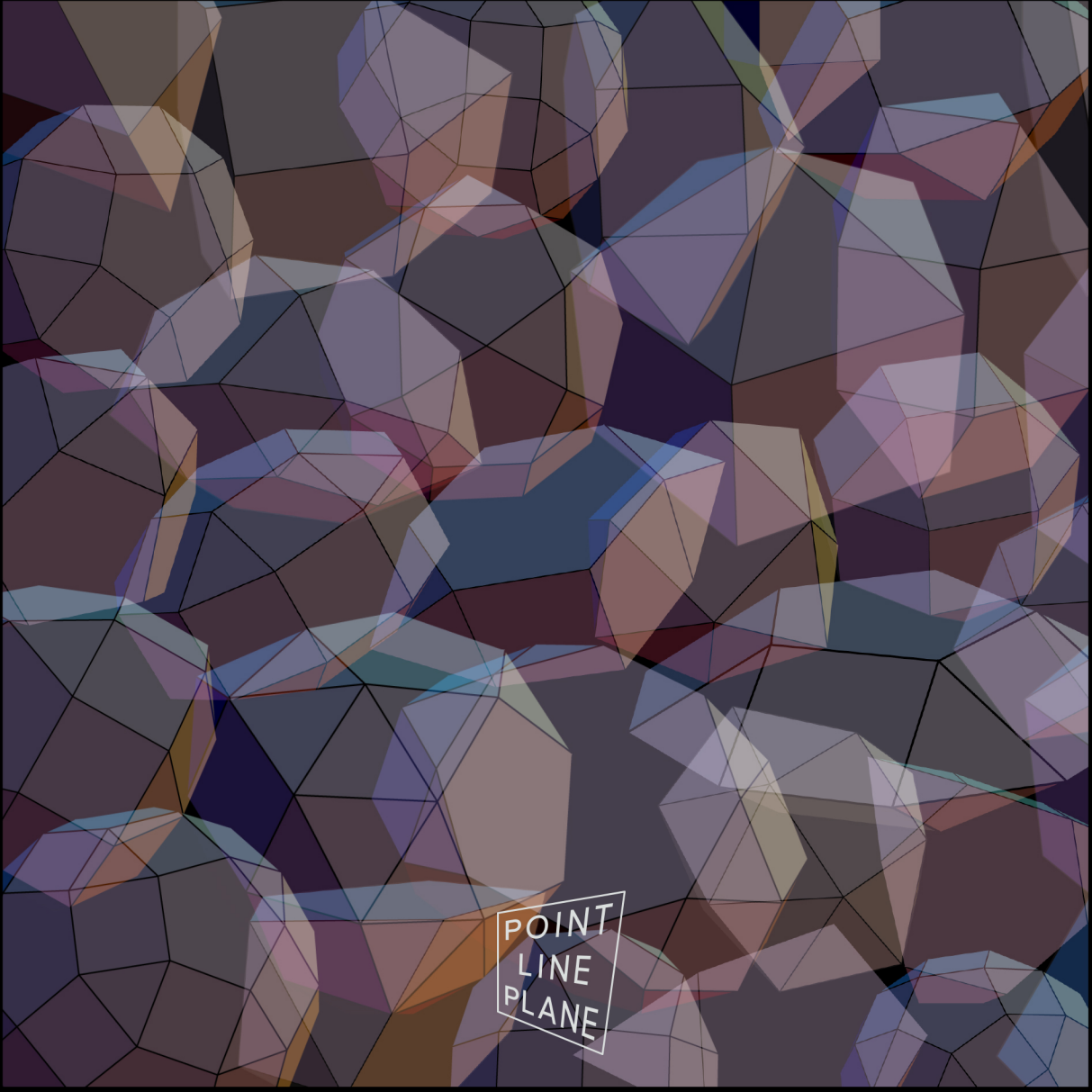
J81
METABIDIMINISHED RHOMBICOSIDODECAHEDRON



J82
GYRATE BIDIMINISHED RHOMBICOSIDODECAHEDRON

J83
TRIDIMINISHED RHOMBICOSIDODECAHEDRON


PYRAMIDS
BIPYRAMIDS
OCTAHEDRON
ICOSAHEDRON
PLATONIC
SOLID
JOHNSON SOLIDS
CUPOLAE
ROTUNDA


POINT
LINE
PLANE