

THE NINETY-TWO JOHNSON SOLIDS





Mathematician Norman Johnson was born in Chicago in 1930.

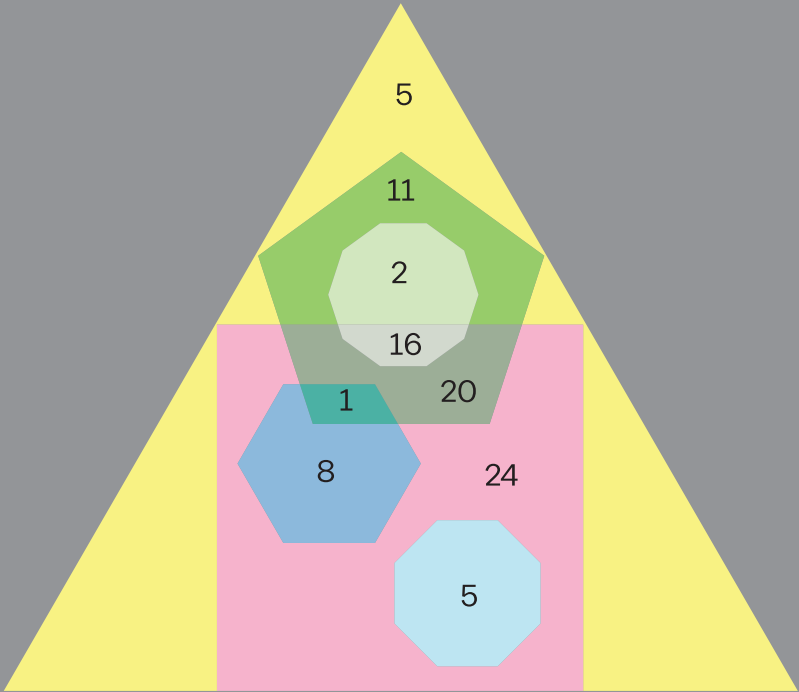
In 1966, he identified 92 specific non-uniform convex solids with regular faces and speculated that no more could exist. After earning his PhD from the University of Toronto that same year, Johnson went on to hold a position at the Mathematics Department of Wheaton College in Massachusetts until his retirement in 1998. His PhD supervisor was the great geometer H.S.M. Coxeter, known for his 1973 book *Regular Polytopes*, which enumerates the properties of perfect solids through higher dimensions.

In 1969, it was proven by mathematician Victor Zalgaller that Johnson's theory was correct. Only 92 possible solids exist. These polyhedra became known as the "Johnson Solids."

Norman Johnson died on July 13, 2017.

Just before his death, he completed the book *Geometries and Transformations*, which was published by Cambridge University Press in 2018.

The 92 Johnson solids are convex polyhedra whose faces are a combination from six different regular polygon shapes. It turns out that only these shapes fit together in a certain number of ways to produce the polyhedra. The polygon faces must be “perfect,” meaning they have symmetry and same-sized sides: an equilateral triangle, a perfect square, and so on. Seven-sided polygons, for example, will not work. There are no Johnson solids composed of all six of the shapes. There are five Johnson solids composed of only triangles. This Venn diagram maps the number of shape combinations that comprise the 92 polyhedra. In the following pages, you will find the Johnson solids organized into groups according to their shapes. On the facing page is a summary.



NUMBER OF SIDES

3 TRIANGLE

4 SQUARE

5 PENTAGON

6 HEXAGON

8 OCTAGON

10 DECAGON

NUMBER OF JOHNSON SOLIDS

5

24

11

20

8

5

2

1

16

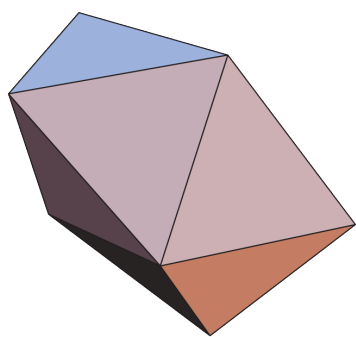
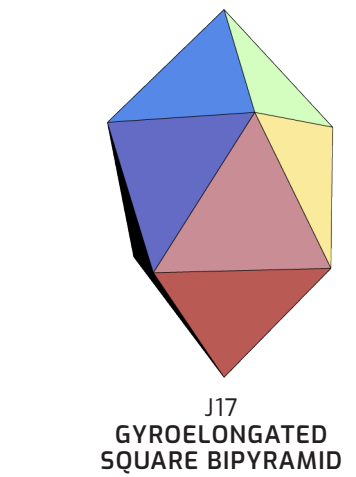
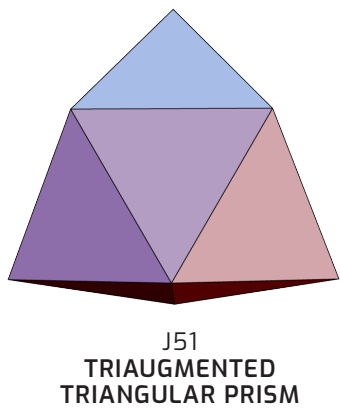
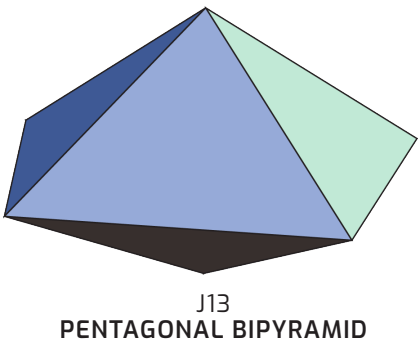
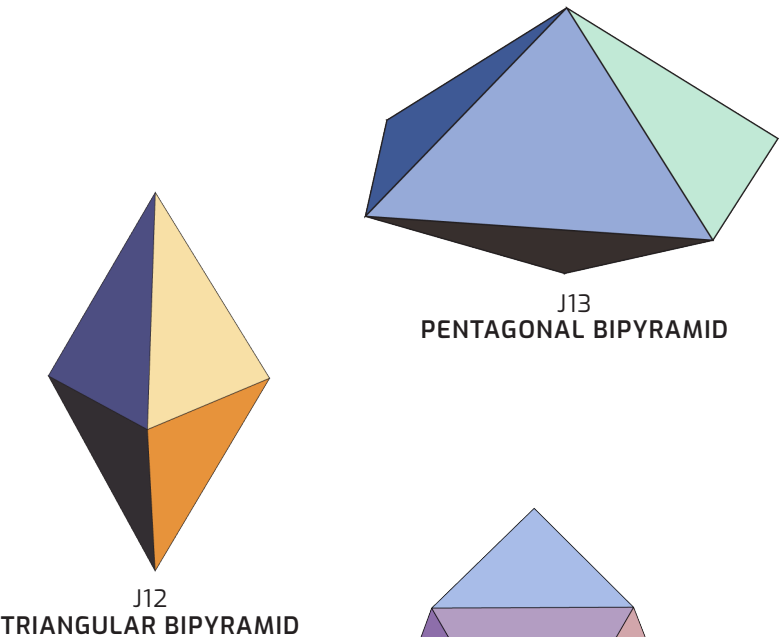
TOTAL = 92

square pyramid § pentagonal pyramid § triangular cupola § square cupola § pentagonal cupola §
pentagonal rotunda § elongated triangular pyramid § elongated square pyramid § elongated pentagonal
pyramid § gyroelongated square pyramid § gyroelongated pentagonal pyramid § triangular bipyramid
§ pentagonal bipyramid § elongated triangular bipyramid § elongated square bipyramid § elongated
pentagonal bipyramid § gyroelongated square bipyramid § elongated triangular cupola § elongated
square cupola § elongated pentagonal cupola § elongated pentagonal rotunda § gyroelongated
triangular cupola § gyroelongated square cupola § gyroelongated pentagonal cupola § gyroelongated
pentagonal rotunda § gyrobifastigium § triangular orthobicupola § square orthobicupola § square
gyrobicupola § pentagonal orthobicupola § pentagonal gyrobicupola § pentagonal orthocupolarotunda
§ pentagonal gyrocupola rotunda § pentagonal orthobiotunda § elongated triangular orthobicupola
§ elongated triangular gyrobicupola § elongated square gyrobicupola § elongated pentagonal
orthobicupola § elongated pentagonal gyrobicupola § elongated pentagonal orthocupolarotunda
§ elongated pentagonal gyrocupolarotunda § elongated pentagonal orthobiotunda § elongated
pentagonal gyrobirotunda § gyroelongated triangular bicupola § gyroelongated square bicupola §
gyroelongated pentagonal bicupola § gyroelongated pentagonal cupolarotunda § gyroelongated
pentagonal birotunda § augmented triangular prism § biaugmented triangular prism § triaugmented
triangular prism § augmented pentagonal prism § biaugmented pentagonal prism § augmented
hexagonal prism § parabiaugmented hexagonal prism § metabiaugmented hexagonal prism §
triauxmented hexagonal prism § augmented dodecahedron § parabiaugmented dodecahedron §
metabiaugmented dodecahedron § triaugmented dodecahedron § metabidiminished icosahedron §
tridiminished icosahedron § augmented tridiminished icosahedron § augmented truncated tetrahedron
§ augmented truncated cube § biaugmented truncated cube § augmented truncated dodecahedron §
parabiaugmented truncated dodecahedron § metabiaugmented truncated dodecahedron § triaugmented
truncated dodecahedron § gyrate rhombicosidodecahedron § parabigyrate rhombicosidodecahedron
§ metabigyrate rhombicosidodecahedron § trigyrate rhombicosidodecahedron § diminished
rhombicosidodecahedron § paragyrate diminished rhombicosidodecahedron § metagyrate diminished
rhombicosidodecahedron § bigyrate diminished rhombicosidodecahedron § parabidiminished
rhombicosidodecahedron § metabidiminished rhombicosidodecahedron § gyrate bidiminished
rhombicosidodecahedron § tridiminished rhombicosidodecahedron § snub disphenoid § snub square
antiprism § sphenocorona § augmented sphenocorona § sphenomegacorona § hebesphenomegacorona
§ disphenocingulum § bilunabiotunda § triangular hebesphenorotunda

THE SOLIDS

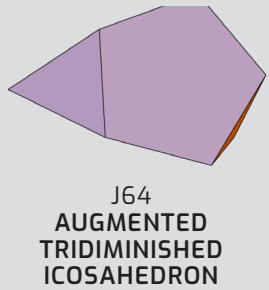
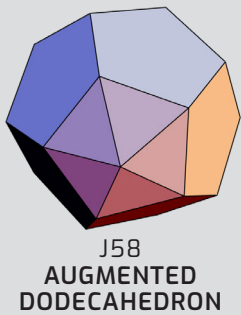
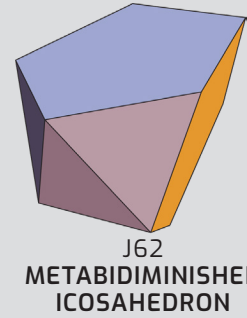
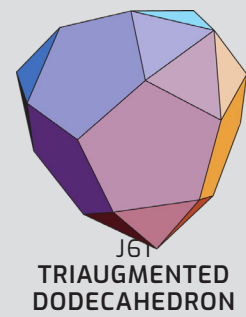
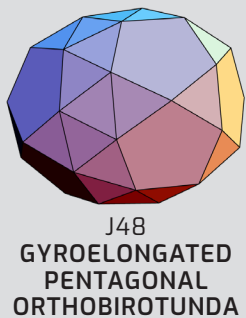
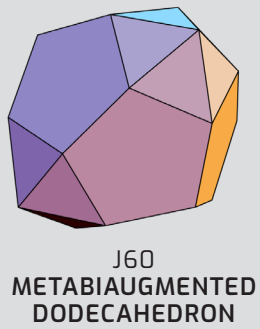
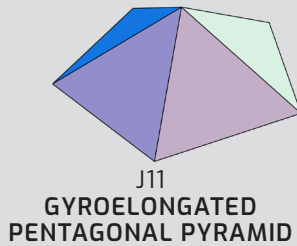
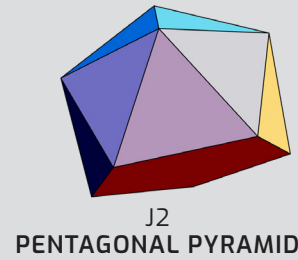
Triangle-only faced

Five solids are called *deltahedra*, meaning they are made of only triangles. Johnson solid triangles are always equilateral.



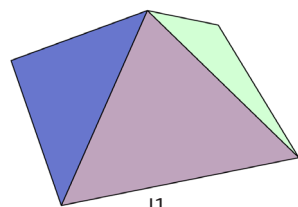
Triangle + Pentagon faced

Eleven solids have only triangle and pentagon faces.

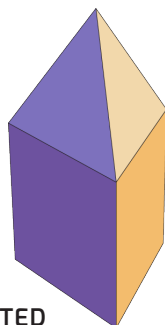


Triangle + Square faced

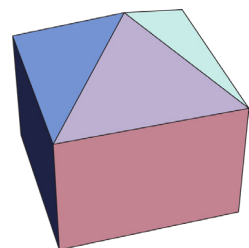
Twenty-four solids have faces made up of only triangles and squares.



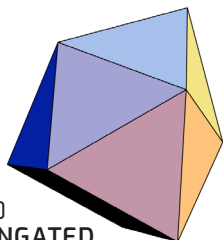
J1
SQUARE PYRAMID



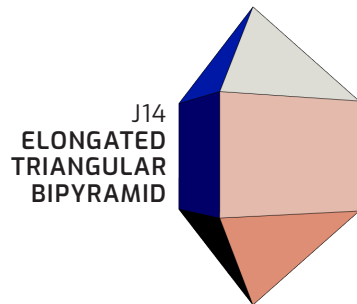
J7
ELONGATED
TRIANGULAR PYRAMID



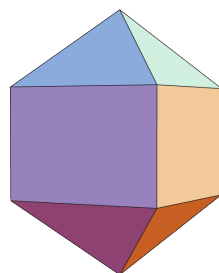
J8
ELONGATED
SQUARE
PYRAMID



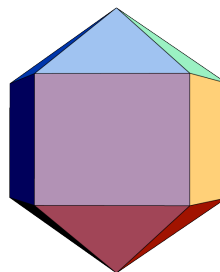
J10
GYROELONGATED
SQUARE PYRAMID



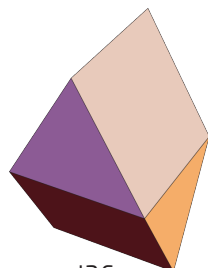
J14
ELONGATED
TRIANGULAR
BIPYRAMID



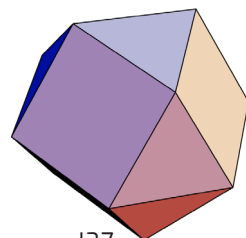
J15
ELONGATED
SQUARE
BIPYRAMID



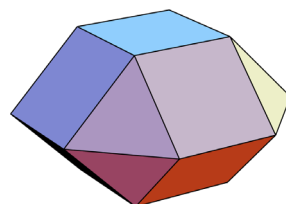
J16
ELONGATED
PENTAGONAL
BIPYRAMID



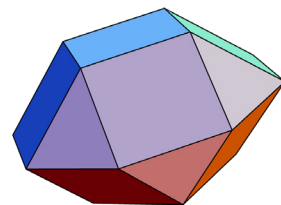
J26
GYROBIFASTIGIUM



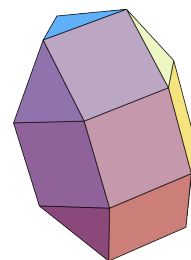
J27
TRIANGULAR
ORTHOBICUPOLA



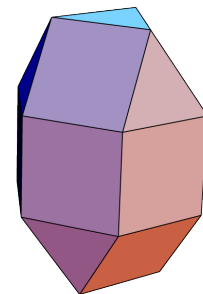
J28
SQUARE
ORTHOBICUPOLA



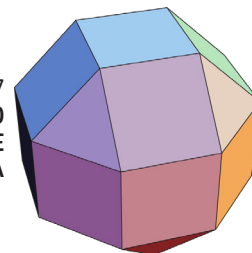
J29
SQUARE
GYROBICUPOLA



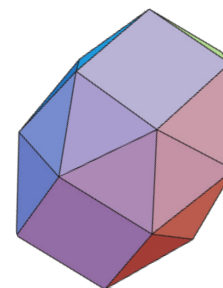
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ELONGATED
TRIANGULAR
ORTHOBICUPOLA



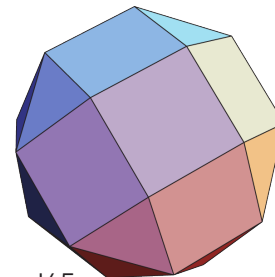
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ELONGATED
TRIANGULAR
GYROBICUPOLA



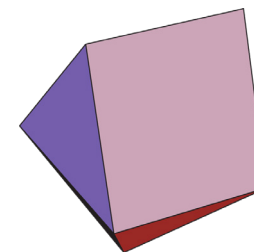
J37
ELONGATED
SQUARE
GYROBICUPOLA



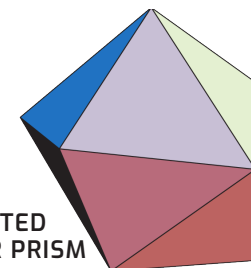
J44
GYROELONGATED
TRIANGULAR
BICUPOLA



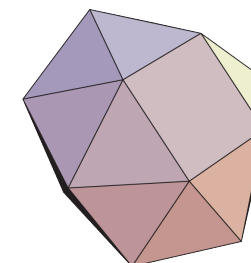
J45
GYROELONGATED
SQUARE BICUPOLA



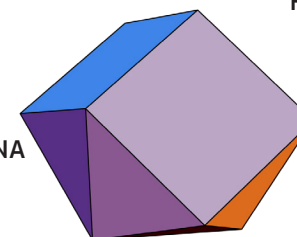
J49
AUGMENTED
TRIANGULAR PRISM



J50
BIUGMENTED
TRIANGULAR PRISM



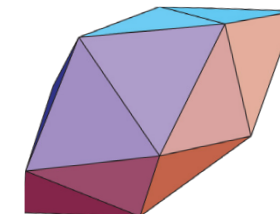
J85
SNUB SQUARE
ANTIPRISM



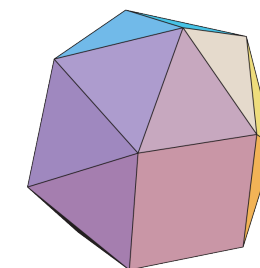
J86
SPHENOCORONA



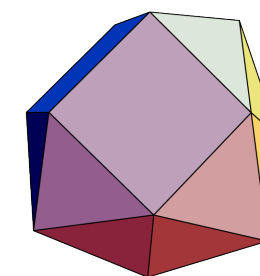
J87
AUGMENTED
SPHENOCORONA



J88
SPHENOMEGACORONA



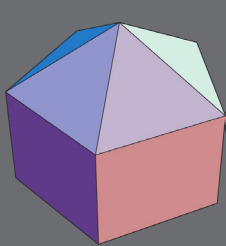
J89
HEBESPHENOMEGACORONA



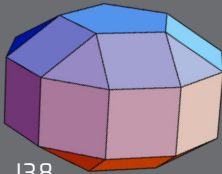
J90
DISPHENOCINGULUM

Triangle + Square + Pentagon faced

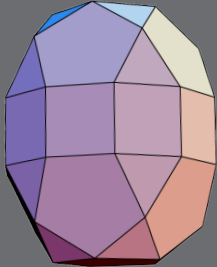
Twenty solids have only triangle, square and pentagon faces



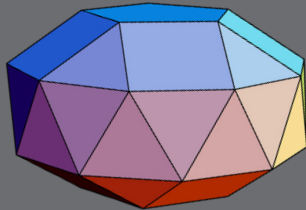
J9
ELONGATED
PENTAGONAL
PYRAMID



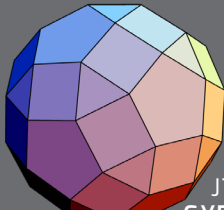
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ELONGATED
PENTAGONAL
ORTHOBIKUPOLA



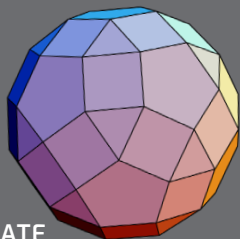
J42
ELONGATED
PENTAGONAL
ORTHOBIROTUNDA



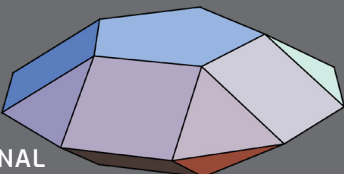
J46
GYROELONGATED
PENTAGONAL
BICUPOLA



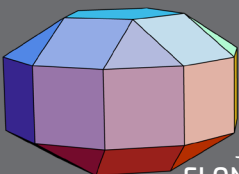
J72
GYRATE
RHOMBICOSIDODECAHEDRON



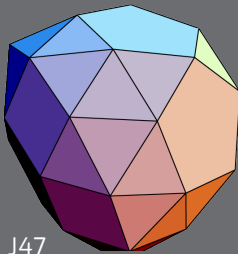
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PARABIGYRATE
RHOMBICOSIDODECAHEDRON



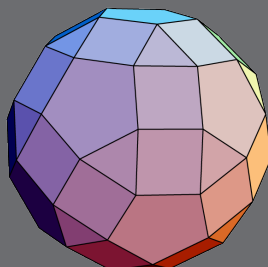
J30
PENTAGONAL
ORTHOBIKUPOLA



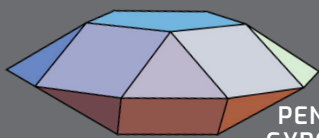
J39
ELONGATED
PENTAGONAL
GYROBIKUPOLA



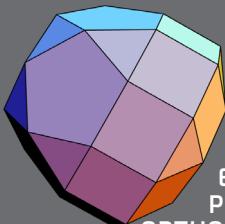
J47
GYROELONGATED
PENTAGONAL
CUPULAROTUNDA



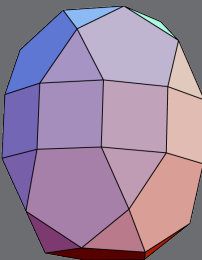
J74
METABIGYRATE
RHOMBICOSIDODECAHEDRON



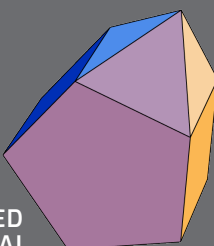
J31
PENTAGONAL
GYROBIKUPOLA



J40
ELONGATED
PENTAGONAL
ORTHOCUPULAROTUNDA

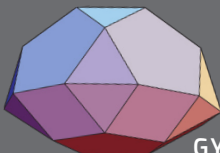
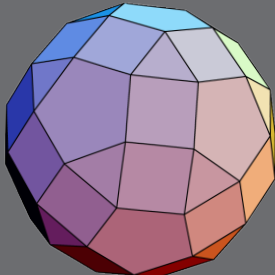


J43
ELONGATED
PENTAGONAL
GYROBIROTUNDA

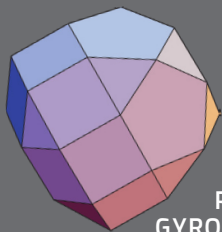


J52
AUGMENTED
PENTAGONAL
PRISM

J75
TRIGYRATE
RHOMBICOSIDODECAHEDRON

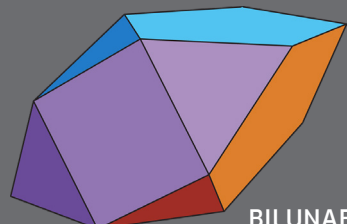
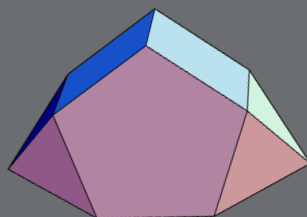


J33
PENTAGONAL
GYROCUPULAROTUNDA



J41
ELONGATED
PENTAGONAL
GYROCUPULAROTUNDA

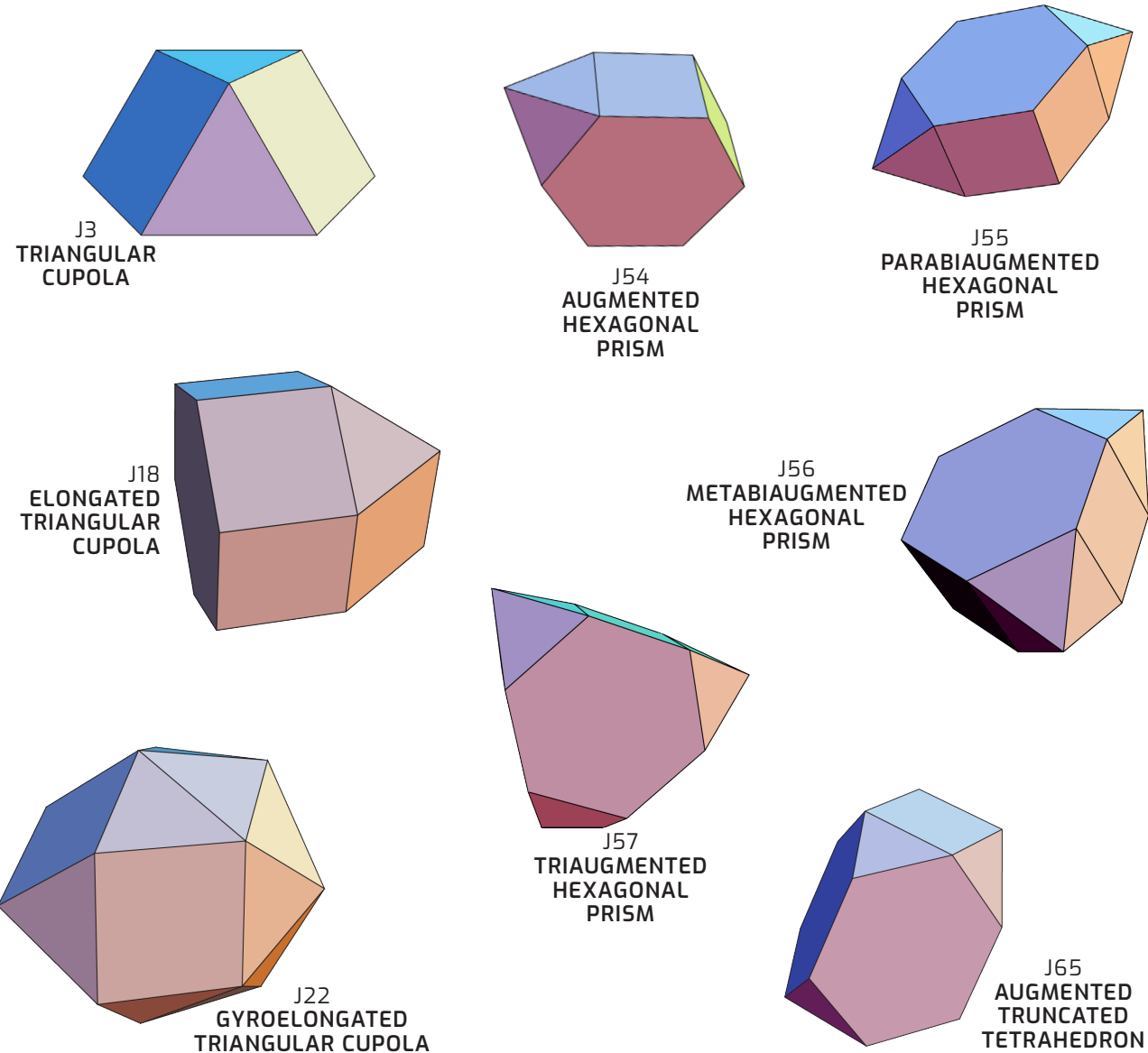
J53
BIAUGMENTED
PENTAGONAL
PRISM



J91
BILUNABIROTUNDA

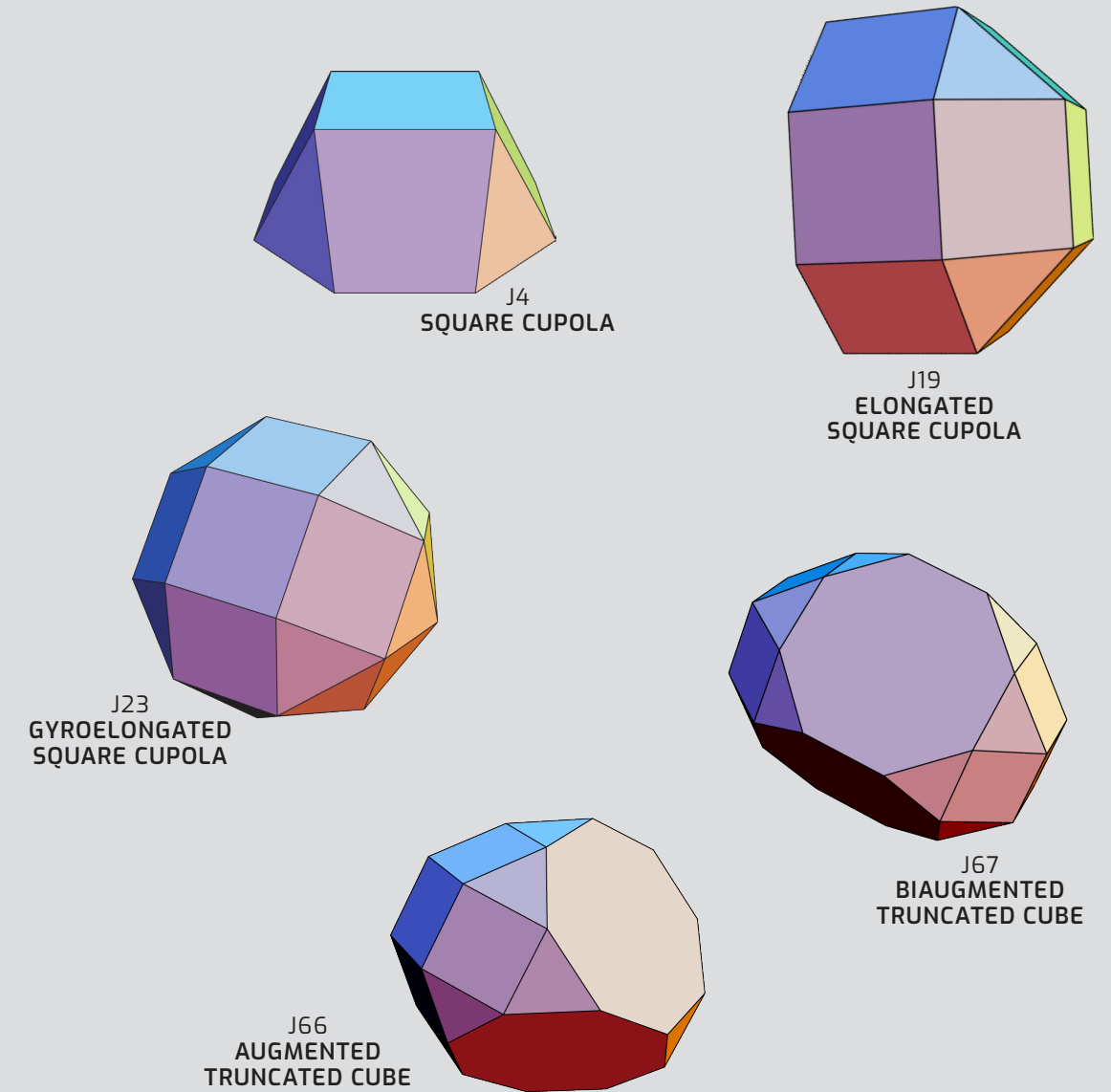
Triangle + Square + Hexagon faced

Eight solids have only triangle, square and hexagon faces



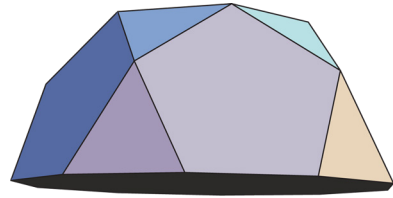
Triangle + Square + Octagon faced

Five solids have only triangle, square and octagon faces

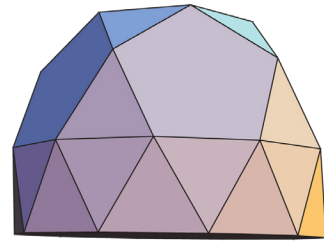


Triangle + Pentagon + Decagon faced

Two solids have only triangle, pentagon and decagon faces



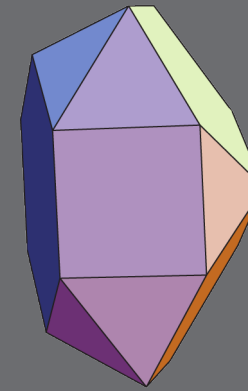
J6
PENTAGONAL
ROTUNDA



J25
GYROELONGATED
PENTAGONAL
ROTUNDA

Triangle + Square + Pentagon + Hexagon faced

Only one solid has faced composed of triangle, square, pentagon and hexagon

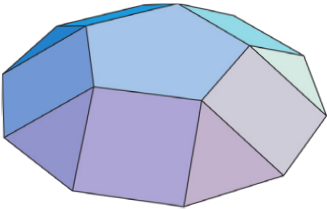


J92
TRIANGULAR
HEBESPHENOROTUNDA

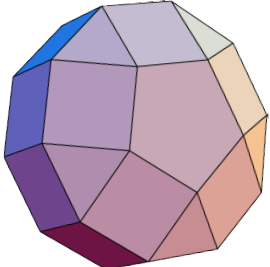
Triangle + Square + Pentagon + Decagon faced

Sixteen solids have only triangle, square, pentagon and decagon faces

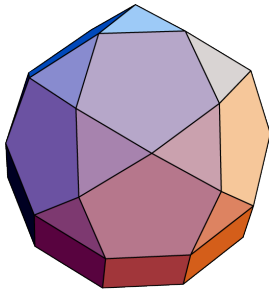
J5
PENTAGONAL
CUPOLA



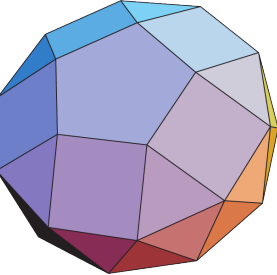
J20
ELONGATED
PENTAGONAL
CUPOLA



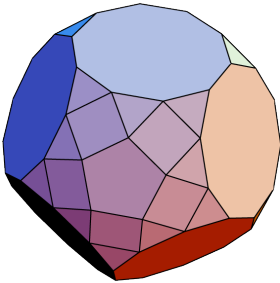
J21
ELONGATED
PENTAGONAL
ROTUNDA



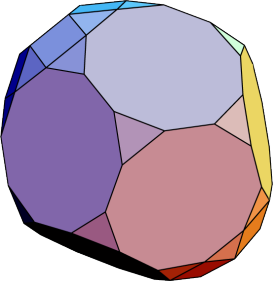
J24
GYROELONGATED
PENTAGONAL
CUPOLA



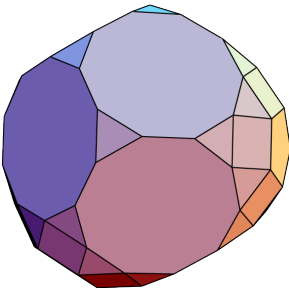
J68
AUGMENTED
TRUNCATED
DODECAHEDRON



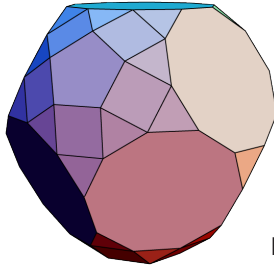
J69
PARABIAUGMENTED
TRUNCATED
DODECAHEDRON



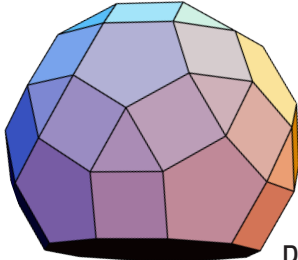
J70
METABIAUGMENTED
TRUNCATED
DODECAHEDRON



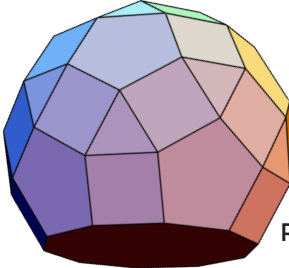
J71
TRIAUGMENTED
TRUNCATED
DODECAHEDRON



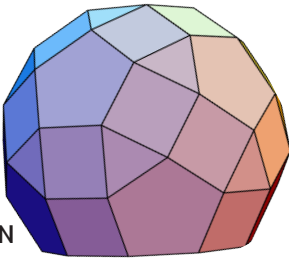
J76
DIMINISHED
RHOMBICOSIDODECAHEDRON



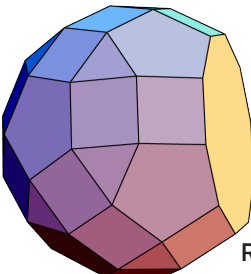
J77
PARAGYRATE
DIMINISHED
RHOMBICOSIDODECAHEDRON



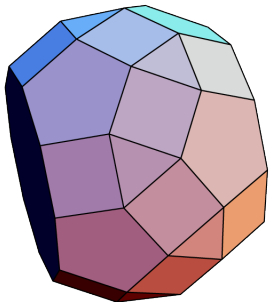
J78
METAGYRATE
DIMINISHED
RHOMBICOSIDODECAHEDRON



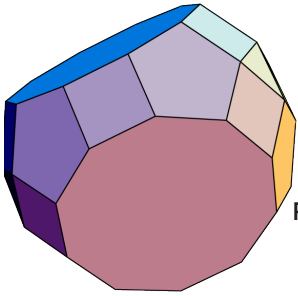
J79
BIGYRATE
DIMINISHED
RHOMBICOSIDODECAHEDRON



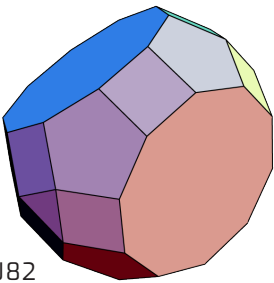
J80
PARABIDIMINISHED
RHOMBICOSIDODECAHEDRON



J81
METABIDIMINISHED
RHOMBICOSIDODECAHEDRON



J82
GYRATE BIDIMINISHED
RHOMBICOSIDODECAHEDRON



J83
TRIDIMINISHED
RHOMBICOSIDODECAHEDRON

